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TESIS DOCTORAL

Essays on Financial Frictions Occupational Choice and Trade

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Autor: *Lian Allub*

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Getafe, de de

"No te des por vencido, ni aun vencido,
no te sientas esclavo, ni aun esclavo;
trémulo de pavor, piénsate bravo,
y arremete feroz, ya mal herido."

Almafuerte

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Abstract

My dissertation consists of two chapters where I study the role of financial frictions and openness to trade on several macroeconomic variables such as gross domestic product or total factor productivity.

In the first chapter, I study the effects that trade and multinational production barriers have on countries of different size. The gains from openness to trade and multinational production (MP) depend largely on country size. A large country may attract more foreign firms by closing itself to trade, while a small country may attract a larger amount of MP if trade costs with its neighbors are low because it can be used as an export platform. I develop a model to study these effects, where firms face non-convex decisions of whether to serve a foreign country by exporting from the home country, exporting from a third country, or producing in the foreign country. I calibrate the model separately for South America and Europe. I find that the gains from openness in Europe double those in South America, and that the distribution of these gains varies less with size in South America. I also find that MP is more important in explaining the gains from openness in large countries, but the export platform mechanism is more important in small countries. Finally, I find that trade and MP have important implications for the size distribution of firms.

In the second chapter, which is joint with Andrés Erosa, we study the effects of financial frictions on occupational choice decisions and on economic inequality. To address this question, we develop a quantitative theory of entrepreneurship, income inequality, and financial frictions disciplined with household data from Brazil. Our theory extends Lucas (1978) by modeling heterogeneity in two skills: −working and managerial skills. Consistently with the evidence, the theory implies three occupational categories: workers, employers, and self-employed entrepreneurs. We find that the removal of financial frictions decreases self-employment rates from 24% to 11% (with small effects on the number of employers), increases aggregate output by 48%, and has non-trivial effects on the distribution of income. We also find that while most households benefit from a reform that eliminates enforcement problems, the majority of employers (about two thirds) lose from the reform. By depressing the demand for labor, limited enforcement depresses the equilibrium wage rate, increasing the profits of employers. Our theory thus suggests that employers in Brazil may have a vested interested in maintaining a status quo with low enforcement.

Resumen

Mi tesis doctoral consiste de dos capítulos en donde estudio los efectos que tienen la fricciones financieras y el comercio internacional sobre diferentes variables macroeconómicas, tales como el producto bruto interno o la productividad total de los factores.

En el primer capítulo, estudio los efectos que tienen las barreras al comercio y a la producción multinacional en países de distintos tamaños. Las ganancias de abrirse al comercio internacional y a la producción multinacional dependen del tamaño de un país. Un país grande puede atraer más firmas extranjeras cerrándose al comercio internacional, mientras que un país pequeño puede atraer una mayor cantidad de firmas extranjeras bajando los costes de comerciar con países vecinos al poder ser utilizado como una plataforma de exportación. Para estudiar estos mecanismos, construyo un modelo donde las firmas enfrentan costes no convexos de vender sus bienes mediante exportaciones, instalando una planta en el país de destino, o exportando desde un tercer país. Calibro el modelo de forma separada para América del Sur y Europa. El primer resultado es que las ganancias que los países de Europa obtienen de su actual nivel de apertura duplican las ganancias que obtienen los países de Sudamérica, y a su vez estas ganancias varían mas con el tamaño del país que en el caso de Sudamérica. También encuentro que la producción multinacional es más importante para explicar las ganancias de apertura en los países grandes que en los países pequeños, pero el mecanismo de actuar como plataforma de exportación tiene un rol más importante en los países pequeños. Finalmente, encuentro que tanto el comercio internacional como la producción multinacional tienen implicaciones importantes en la distribución del tamaño de las firmas.

En el segundo capítulo, co-autoreado con Andrés Erosa, estudiamos los efectos de las fricciones financieras en las decisiones ocupacionales y en la desigualdad. A fin de analizar este tema, desarrollamos una teoría cuantitativa de emprendedurismo, desigualdad del ingreso y fricciones financieras, disciplinada con datos de Brasil. Nuestra teoría extiende el modelo presentado en Lucas (1978), modelando heterogeneidad en dos habilidades: habilidad como trabajador y habilidad como *manager*. Consistente con la evidencia, la teoría implica tres categorias ocupacionales: trabajadores, cuentapropistas y empleadores. Encontramos que el remover las fricciones financieras disminuye la proporción de cuentapropistas de 24% a 11% (con cambios pequeños en el número de empleadores), incrementa el producto en un 48%, y tiene efectos no triviales en la distribución del ingreso. También encontramos que mientras la mayoría de las familias se beneficia de la reforma que elimina las fricciones financieras, la mayoría de los empleadores (alrededor de dos tercios) pierde con esta reforma. Disminuyendo la demanda de trabajo, las fricciones financieras disminuyen el salario de equilibrio e incrementan el beneficio de los empleadores. Nuestra teoría sugiere que los empleadores en Brasil pueden tener interés en mantener el *status quo* con fricciones financieras.

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Chapter 1

Asymmetric Effects of Trade and FDI: South America versus Europe

1.1 Introduction

The gains from openness to trade and multinational production (MP) depend largely on country size. A large country may attract more foreign firms by closing itself to trade, while a small country may attract a larger amount of MP if trade costs with its neighbors are low because it can be used as an export platform. I develop a quantitative theory to assess how trade barriers and country size interact to determine the location of multinational firms, and its effects on GDP, GNP, and firm size distribution.

Trade barriers affect the location decision of multinational firms in two ways. First, trade barriers change the relative cost of exporting compared to producing in the consumption location. A firm may decide to become multinational if it is cheaper to serve a market by MP rather than by exporting. Second, trade barriers change the relative cost of exporting from two different locations. Firms may use a country as an export platform to serve a set of neighbor countries.[1] The importance of these two channels depends critically on country size. For a small country it is difficult to attract multinational production to overcome trade barriers since its domestic market is small. Then, for a small country the way of attracting multinational firms is by offering the possibility of serving other countries, i.e. to be used as an export platform. On the other hand, large countries, as they have large markets, can attract MP even with high trade costs (they may attract even a larger amount of MP with trade barriers than without).

I quantitatively compare the performance of large and small countries in two different regions: South America, with high trade barriers; and Europe, with low trade barriers. In order to

[1] I call this mechanism bridge multinational production following Ramondo and Rodríguez-Clare (2013).

do so, I use a heterogeneous firm model of trade with monopolistic competition, asymmetric countries, and allowing for MP and bridge multinational production (BMP). I perform two separate calibrations, one for each region. In the calibration for South America, I include Argentina, Brazil, Chile and Uruguay. In the calibration for Europe, I include four members of the European Union (France, Italy, Netherlands and United Kingdom). In both cases I include a fifth country which stands for the rest of the world. To calibrate the model I use data on bilateral trade flows, bilateral FDI flows, firm composition (domestic and foreign, exporters and non-exporters), GDP per capita, manufacturing trade deficit, and labor force size.

To assess the gains from openness I compare the real GDP in the calibrated model economies with the real GDP in autarky. I find that the gains from openness in Europe double those in South America (10.5% versus 5.3% of real GDP), indicating that, as a region, South America is closed, benefitting little from trade and MP. Then, I perform three experiments to disentangle the contribution of the different channels through which MP affects the gains from openness: (i) by producing and selling in the domestic country (MP itself); (ii) by using the domestic country as an export platform (BMP); and (iii) both effects together. To asses the contribution of MP itself (without considering the BMP channel), I compare the losses of going to autarky in a world with MP and no BMP versus a world without MP (this implies no BMP neither). I find that MP itself plays a bigger role in large countries. In the Netherlands the losses of going to autarky in a world without BMP are 20% larger than in a world without MP, while in Italy are 84% larger. This means that for the Netherlands most of the gains from MP come from BMP while in Italy most of the gains come from MP itself (not from BMP). To assess the role played by BMP, in the second experiment I compare the losses of going to autarky in the calibrated model economies with the losses in a world in which BMP is not allowed. I find that BMP is more important for small countries. For example, in the Netherlands the losses of going to autarky are 20% higher in the baseline economy than in the world without BMP, while in Italy are only 10% higher. Finally, to assess the contribution of MP (including both mechanisms), I compare the losses of going to autarky in the calibrated model economies with the losses of going to autarky in a world without MP. I find that MP as a whole is more important for large countries. The losses of going to autarky Italy are 100% larger than in a world without MP, while in the Netherlands are only 44% larger. It is worth mentioning that in South America, since economies are more closed (reducing the possibility of exploiting BMP) and foreign firms are much less efficient (reducing the gains from MP itself)[2], the role played by MP and BMP in explaining the gains from openness in large and small countries is changed. Uruguay (the small country) benefits more from BMP than Brazil, but MP itself is equally important in both countries.

[2]Even though I do not explicitly model this aspect, the low efficiency of multinationals (high γ in the model) may be due to institutions, labor market policies, input quality, etc.

My second set of finding is that the differences between what a small and what a big country lose when going to autarky are very different among regions. In South America, losses are more homogeneously distributed (vary less with size) than in Europe. The difference between what Brazil (the largest country) loses in real manufacturing GDP and what Uruguay (the smallest country) loses is of 8.5 percentage points, while in Europe this difference is of 14.7 percentage points. The higher heterogeneity in Europe comes from the fact that Europe is more opened than South America which allows a small country in Europe, like the Netherlands, to take more advantage from trade and MP than a small country in South America, like Uruguay.

Next, I study what would be the gains in South America of improving the degree of openness. South America is a much more closed region and there could be large gains from openness. To do this, I decrease the variable trade cost in all countries in the calibration for South America setting them to the average level in Europe.[3] I find that all countries benefit from this reduction, but the smallest country, Uruguay, is the one who benefits the most with an increase in manufacturing real GDP of 30%. If, in addition, the efficiency of multinational firms operating in these countries increases 20%, the gains would increase from 30% to 50%. The gains for Uruguay would be even larger if the improvement in efficiency takes place only in Uruguay but not in the rest of countries. This is because in this case Uruguay would face less competition to attract multinational firms. However BMP is crucial to attain the gains from better efficiency. If I do not allow for BMP, the additional gains Uruguay would get by improving efficiency (on top of the ones obtained by reducing trade costs) are close to zero.

Finally, I study how openness affects the size distribution of businesses across countries and regions. In the calibrated model economies, the size distribution of firms changes across countries of different size in the same region, and also across countries of the same size in different region. I find that openness increases the proportion of large firms (with more than 250 employees) more in small than in large countries, and also that this effect is larger in the open than in the closed region. In the baseline economy, the Netherlands has 4.2% of large firms while Uruguay has 1.1%, Italy has 1.7% and Brazil has 0.8%. Therefore, internationalization of firms has an important effect on the size distribution of firms. This is, I believe, an important contribution to the misallocation literature on business size distribution. Previous papers have studied the effects of size dependent policies (Guner, Ventura, and Xu (2008), Restuccia and Rogerson (2008), García-Santana and Pijoan-Mas (2012)), capital market imperfections (Erosa (2001), Amaral and Quintin (2010), Buera, Kaboski, and Shin (2011), Greenwood, Sanchez, and Wang (2010)) and trade (Melitz (2003), Piguillem and Rubini (2012)) on firm size distribution. My paper contributes to this literature by addressing the effect that trade and MP have on the size of businesses.

[3] I understand that in the trade barriers I use in the model, I include features like geography or language which vary a lot between regions and are probably not subject to reductions. Still using Europe as the best scenario that South America can reach is a very informative exercise on the size of the gains that could be obtained.

Previous studies have analyzed the interaction of trade and MP but these studies did not allow for BMP.[4] Recently, three papers model BMP. Ramondo and Rodríguez-Clare (2013) use a ricardian model of trade to address the gains from openness (trade and MP). However, they cannot address the effects of country size on the location of multinational firms since they assume perfect competition and as a result they do not model fixed costs of MP. Arkolakis, Ramondo, Rodriguez-Clare, and Yeaple (2013) model trade and MP with monopolistic competition. However, they do not include fixed costs of setting up foreign firms. Fixed costs are important to study the role that the size of a country plays in determining the location of multinationals which is the goal of my paper. With fixed costs there are increasing returns in production, which makes the size of a market an important variable to make a location decision. The closest paper to mine is Tintelnot (2012). He includes fixed cost of producing and performing MP and studies the gains from openness (trade and MP) in a monopolistic competition set-up. The focus in my theory is on how BMP shapes the impact of country size and geography (the distribution of trade costs across different countries) on the determination of output and trade across countries. In particular, I use my theory to quantitatively assess and compare the geography of trade and multinational production barriers in South America versus Europe. Finally, I also assess the effects of trade, MP and BMP on the distribution of firm sizes.

1.2 Facts

I now document some facts on the relation between trade, FDI and size for South America and Europe.[5] It is a well known fact in the trade literature that small countries benefit the most from trade. As a result it is expected to observe that small countries are more open than large countries. Figure 1.1 presents the relation between trade and size for South America (blue circles) and Europe (orange squares).[6,7] First, note that the points for Europe are above those for South America which indicates that for countries of similar sizes, trade-to-GDP ratio is larger in Europe than in South America. Then European countries are more open and benefit more from trade. Also the fitted line is steeper in Europe than in South America (the slopes are -0.51 for Europe and -0.28 for South America) which indicates that trade-to-GDP varies more with size in Europe than in South America. Then European countries are more open than South American countries, and the difference increases with country size, suggesting that the extent to which small countries benefit from trade in South America relative to Europe is much lower than that for large countries.

[4]See Helpman (1984), Horstmann and Markusen (1992), Markusen and Venables (2000), Helpman, Melitz, and Yeaple (2004), Irarrazabal, Moxnes, and Opromolla (2013)

[5]The data is from the United Nations Conference on Trade and Development (UNCTAD) Statistics for the year 2012.

[6]Trade is measured as trade in merchandising over GDP and size as the natural logarithm of population.

[7]I exclude Cyprus and Luxembourg from the European Union sample.

Figure 1.2 presents the relation between FDI and size for the same group of countries.[8] In this case we observe that for Europe there is a negative relation between FDI and size: small countries have larger ratios of FDI/GDP. However, this relation is flat in South America (the slopes of the fitted line are -0.30 for Europe and -0.02 for South America). Then, the extent to which small countries benefit from FDI in South America relative to Europe is lower than that for large countries.

To sum up, South American countries show lower trade-to-GDP ratios than European countries. Also, the relation between trade and size is less steep in South America than in Europe, increasing the differences in the trade-to-GDP ratios between the two regions for small countries. As trade is key in small countries to attract foreign firms (to be used as an export platform) we expect small South American countries to not be able to attract as much FDI as small European countries. This is confirm in Figure 1.2 where the relation between FDI and size is negative in Europe but it is flat for South America. The lower trade-to-GDP and FDI-to-GDP ratios in South America, indicates that the region as a whole is more closed than Europe with small countries benefiting less from the potential gains from openness. In what follows I develop a model of trade and multinational production to measure trade and MP barriers that countries are facing and assessing the potential gains from openness that South American countries may obtain.

1.3 Model

The model builds on Melitz (2003) but adds the possibility of multinational production and bridge multinational production. There is a set of countries with different sizes. In each country there is a representative consumer. In the world economy there are two types of goods: a homogeneous good and a differentiated good, both of them tradable. Each differentiated good is produced by a firm with a given productivity. Differentiated goods have three sub-indices: the first one indicates where the good is consumed, the second one where the good is produced and the last one to which country the firm that produced the good belongs. For example, $q_{ijk}(\omega)$ is the quantity of good ω consumed in country i and produced in country j by a firm from country k.

1.3.1 Countries

The world economy consists of $i = 1, ..., N$ countries; two sectors: a homogeneous-good sector (sector 0) and a differentiated good sector (sector 1); one factor of production, labor; and a continuum of goods indexed by $\omega \in \Omega$. All goods in the economy are tradable. Each country

[8] FDI is measured as the stock of FDI in a country as a proportion of GDP.

has a population of L_i individuals who supply labor inelastically. Let w_i be the wage in country i in terms of the homogeneous good. I set the price of the homogeneous good, P_0, to be the numeraire. In each country there is a large mass of potential firms producing.

1.3.2 Consumers

In each country there is a representative consumer with Cobb-Douglas preferences:

$$U_i = q_{i,0}^{\mu_0} q_{i,1}^{(1-\mu_0)} \ , \tag{1.1}$$

where μ_0 is the share of the homogeneous good in total consumption and q_1 is a Dixit-Stiglitz aggregator[9] :

$$q_{i,1} = \left(\int q_i(\omega)^{\frac{(\sigma-1)}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}} \ ,$$

where $\sigma > 1$ is the elasticity of substitution between varieties and $q_{i,1}$ are all the varieties consumed in country i.

The above utility function implies that the representative consumer will spend μ_0 share of his income on the homogeneous good and $1-\mu_0$ in differentiated goods. Then the demand functions are:

$$\begin{aligned} q_{i0} &= \frac{\mu_0 E_i}{P_i^0} \ , \\ q_{i1} &= \frac{(1-\mu_0)E_i}{P_i^0} \ , \end{aligned} \tag{1.2}$$

where P_i^0 is the aggregate price index in country i including the homogeneous good sector and E_i is the aggregate expenditure in country i.

Define the expenditure in the differentiated good sector as $(1-\mu_0) * E = E^1$, where E is total expenditure. Then, the demand for variety ω is given by:

$$q_{jki}(\omega) = \frac{E_j^1}{P_j} \left(\frac{p_{jki}(\omega)}{P_j} \right)^{-\sigma} \ , \tag{1.3}$$

where E_j^1 is the aggregate expenditure of country j in differentiated goods and P_j is the aggregate price in the differentiated good sector in country j.[10] The demand of good $q_{jki}(\omega)$ is increasing in total expenditure and the aggregate price of the country where the good is consumed (E_j^1 and P_j), and decreasing in the price of the good.

[9] Where $\rho = \frac{\sigma-1}{\sigma}$. I will use σ or ρ in my definitions depending on convenience.

[10] I will give a formula for P_j later since I still need to define some concepts used in the definition of P_j.

1.3.3 Homogeneous good

Each country has an exogenous endowment z_i of the homogeneous good. This good is traded without any cost. This implies that the price of this good will be equalized among countries. We will denote the price of the homogeneous good as p_0. Each country will be an exporter or importer of this good depending on whether the domestic supply of the good is bigger or smaller than the domestic demand of the good.

Without the homogenous good, the model would require trade imbalances to be compensated by capital account imbalances to get a balanced current account. This would imply that a country having trade deficit would have capital account surplus. Capital account surplus in this model means that profits from domestic firms producing abroad are larger than profits from foreign firms producing in the domestic country. Introducing the homogenous good sector allows the model to have countries with both trade deficit in the differentiated good and also capital account deficits, something that is present in the Latin-American countries I am considering.

1.3.4 Differentiated good sector

1.3.4.1 Production

To produce the differentiated good the only input used is labor.[11, 12] Firms pay a fixed entry cost κ_i^e to make the labor productivity draw ϕ, denominated in labor units (then what a firm pays is $w_i * \kappa_i^e$). I assume that productivities are drawn from a Pareto distribution. After observing the productivity, firms decide whether to produce or not.

Firms have to pay a fixed cost of operation κ_i^d, also denominated in labor units, to produce domestically. There are other activities that the firms can choose to perform: to export, to produce in a foreign country to sell there, or to produce in a foreign country and export from there.

To export firms have to pay a fixed cost (independent of the selling destination) and an iceberg type cost which is partner specific. Firms producing in country i and exporting to country j pay a fixed cost $w_i * \kappa_i^x$ and an iceberg cost τ_{ji} per unit sold i.e. they have to send $\tau_{ji} \geq 1$ units of the good for one unit to arrive destiny.

[11] As a result, this paper analyzes horizontal FDI in the spirit of Markusen (1995). See Barba Navaretti and Venables (2004) for a review of the literature on FDI (both vertical and horizontal FDI).

[12] The work of Irarrazabal, Moxnes, and Opromolla (2013) model trade with vertical FDI. When assuming that foreign firms use imported intermediate goods as input, we are introducing complementarity between trade and MP. With horizontal FDI and allowing for BMP, MP and trade can be substitutes or complements. Ramondo and Rodríguez-Clare (2013) on the other hand model both vertical and horizontal FDI.

When a firm produces abroad its productivity is shifted by a parameter γ. The new productivity of firm from country i producing in country j is $\hat{\phi} = \frac{\phi}{\gamma_{ji}}$. In addition, a firm from country i producing in country j has to pay a fixed cost $w_j * \kappa_j^{MP}$ which is independent from the source country (all foreign firms producing in country j pay the same fixed cost). κ_j^{MP} includes the domestic cost of producing in country j and an extra cost i.e. $\kappa_j^{MP} \geq \kappa_j^d$.

I will now define the costs and profits for firms performing each activity.

A firm from country i producing q_{jki} units in country k and selling to country j has the following variable cost:

$$c_{jki}(\omega) = \frac{\tau_{jk}\gamma_{ki}w_k}{\phi} q_{jki}(\omega) \ .$$

If $j = k$ then $\tau_{jk} = 1$, and if $k = i$ then $\gamma_{ki} = 1$. Notice that a firm from country i producing in country k and selling to country j, with $i \neq k$ and $j \neq k$, has to pay both costs γ_{ki} and τ_{jk}. Maximizing variable profits of a firm from country i for a given activity,

$$\max_{p(\omega)} \pi = p(\omega)q(\omega) - c(\omega) \ , \tag{1.4}$$

where $q(\omega)$ was defined in equation (1.3), we get the price of a variety, given by:

$$p_{jki}(\omega) = \frac{w_k\gamma_{ki}\tau_{jk}}{\rho\phi} \ . \tag{1.5}$$

As each firm produces a different variety we can substitute, without loss of generality, ω by ϕ. Using expression (1.3) and (1.5) we obtain the revenue associated with each activity.

$$\begin{aligned}
\text{Selling Domestically} \Rightarrow r_{iii}(\phi) &= E_i^1 P_i^{\sigma-1}\left(\frac{\rho\phi}{w_i}\right)^{\sigma-1} \\
\text{Exporting from the home country} \Rightarrow r_{kii}(\phi) &= E_k^1 P_k^{\sigma-1}\left(\frac{\rho\phi}{w_i\tau_{ki}}\right)^{\sigma-1} \\
\text{Doing MP in country } k \Rightarrow r_{kki}(\phi) &= E_k^1 P_k^{\sigma-1}\left(\frac{\rho\phi}{w_k\gamma_{ki}}\right)^{\sigma-1} \\
\text{Doing BMP in } k \text{ to sell in } j \Rightarrow r_{jki}(\phi) &= E_j^1 P_j^{\sigma-1}\left(\frac{\rho\phi}{w_k\gamma_{ki}\tau_{jk}}\right)^{\sigma-1}
\end{aligned} \tag{1.6}$$

The next step is to find what firms are going to perform each activity. A firm will perform an activity as long as the activity is profitable. Let start with firms only selling domestically. A firm will sell domestically if

$$\pi_{iii}(\phi) = \frac{E_i^1 P_i^{\sigma-1}}{\sigma}\left(\frac{\rho\phi}{w_i}\right)^{\sigma-1} - \kappa_i^d w_i \geq 0 \tag{1.7}$$

As profits are increasing with productivities, there will be one productivity (the cut-off productivity) for which profits will be equal to zero. I will denote the domestic productivity cut-off as ϕ^*_{iii}. All firms with productivities higher than ϕ^*_{iii} will sell domestically. Now, firms can also export or produce abroad. Before continuing, let me assume the following:

Assumption 1: *A variety is defined by the country of origin of the firm and the country where the good is produced.*

Assumption 2: *Any firm from country i performing an activity has to pay the domestic cost of producing in i.*

With assumption 1 a firm from Uruguay producing in Uruguay and exporting to Brazil is going to sell a different variety than the same firm producing in Brazil and selling in Brazil. The fact that varieties are determined also by the production location simplifies the solution of the problem allowing me to treat each activity as independent activities.[13,14] In the absence of assumption 1 a firm from Uruguay will have to choose how to serve the Brazilian market (either by producing in Uruguay and exporting, by doing MP in Brazil or by doing MP in a third country and exporting to Brazil) since the variety sold is the same independently from the production location. Then, without assumption 1 there will be more competition between countries for attracting MP, which would increase the importance of the efficiency of multinationals operating in the domestic country (parameter γ). Also, without assumption 1 BMP becomes a more important factor for attracting MP. In the quantitative section I discuss in more details the role of assumption 1.

Assumption 2 ensures that there will be no firms exporting or doing MP and not selling in the domestic country.

The profit for a firm from country i exporting to country k is given by:

$$\pi_{kii}(\phi) = \frac{E^1_k P^{\sigma-1}_k}{\sigma} \left(\frac{\rho\phi}{w_i \tau_{ki}} \right)^{\sigma-1} - \kappa^x_i w_i \tag{1.8}$$

Setting this equation equal to zero, we can find the cut-off productivity (ϕ^*_{kii}) for a firm from country i exporting to country k. To fix ideas, let us keep aside the possibility of MP. Then, we have two possibilities for defining the exporting cut-offs:

Case 1: If all the exporting cut-offs are higher than the domestic cut-off in country i, that is if $\phi^*_{iii} < \phi^*_{kii}$ $\forall k$, then the domestic and the exporting cut-offs are well calculated. Firms with productivities $\phi^*_{iii} < \phi < \phi^*_{kii}$ only sell in the domestic market, while firms with productivities $\phi > \phi^*_{kii}$ sell domestically and export.

[13] Using assumption 1, I can extend the results from Melitz (2003) considering multinational production and BMP just as an additional activity which simplifies the problem.

[14] It can be that in the end activities are not fully independent, but I can compute costs and profits for each activity as if they were fully independent.

Case 2: If at least one exporting cut-off ϕ^*_{kii} is lower than the domestic cut-off ϕ^*_{iii}, then we have to re-calculate cut-offs. Denote K^x_i the set of countries k for which the exporting cut-off (from country i to country k) is lower than the domestic cut-off. For countries $k \in K^x_i$ the exporting cut-off is equal to the domestic cut-off ($\phi^*_{iii} = \phi^*_{kii}$). The marginal firm entering in the domestic market (with productivity ϕ^*_{iii}) makes negative profits selling in the domestic market but these negative profits are compensated by the positive profits obtained by exporting to countries $k \in K^x_i$. Then, the productivity cut-off defined in the marginal entrant (ϕ^*_{iii}) solves the following equation:

$$\pi_{iii}(\phi^*_{iii}) + \sum_{k \in K^x} \pi_{kii}(\phi^*_{iii}) = 0. \tag{1.9}$$

Now let us consider the possibility for MP. Allowing for MP brings new cases for the way the domestic cut-off is defined. The profit for a firm from country i producing and selling in country k (performing MP in country k) is given by:

$$\pi_{kki}(\phi) = \frac{E^1_k P^{\sigma-1}_k}{\sigma} \left(\frac{\rho\phi}{w_k \gamma_{ki}} \right)^{\sigma-1} - \kappa^{MP}_k w_k \tag{1.10}$$

To fix ideas, let us ignore the possibility of exporting. We want to focus on how MP affects the calculation of the domestic cut-off. There are two cases again to consider:

Case 3: If all the MP cut-offs are higher than the domestic cut-off in country i, that is if $\phi^*_{iii} < \phi^*_{kki}\ \forall k$, then the domestic and the MP cut-offs are well calculated. Firms with productivities $\phi^*_{iii} < \phi < \phi^*_{kki}$ only sell in the domestic market, while firms with productivities $\phi > \phi^*_{kki}$ sell domestically and perform MP.

Case 4: If at least one MP cut-off (ϕ^*_{kki}) is lower than the domestic cut-off, then we need to follow similar steps as in case 2. Denote by K^{MP}_{ki} the set of countries (k) for which the MP cut-off in country i (ϕ^*_{kki}) is lower than the domestic cut-off in country i (ϕ^*_{iii}). For countries $k \in K^{MP}_{ki}$ the MP cut-off is equal to the domestic cut-off $\phi^*_{kki} = \phi^*_{iii}$. The marginal firm entering in the domestic market (with productivity ϕ^*_{iii}) makes negative profits selling in the domestic market but these negative profits are compensated by the positive profits obtained by performing MP in countries $k \in K^{MP}_{ki}$. Then, the productivity of the marginal entrant in country i solves the following equation:

$$\pi_{iii}(\phi^*_{iii}) + \sum_{k \in K^{MP}_{ki}} \pi_{kki}(\phi^*_{iii}) = 0 \tag{1.11}$$

If we assume that firms can export and do MP, the procedure is the same. The only difference is that if we have exporting cut-offs and MP cut-offs that are below the domestic cut-off, then the the productivity of the marginal entrant in country i solves the following equation:

$$\pi_{iii}(\phi^*_{iii}) + \sum_{k \in K^x} \pi_{kii}(\phi^*_{iii}) + \sum_{k \in K^{MP}_{ki}} \pi_{kki}(\phi^*_{iii}) = 0 \tag{1.12}$$

Finally, a firm may want to use a third country as an export platform (BMP). The profit for a firm from country i, producing in country k and selling in country j is given by:

$$\pi_{jki}(\phi) = \frac{E^1_j P^{\sigma-1}_j}{\sigma} \left(\frac{\rho\phi}{w_k \gamma_{ki} \tau_{jk}} \right)^{\sigma-1} - \kappa^x_k w_k \tag{1.13}$$

Setting the above equation to zero, we can find the BMP cut-off productivity (ϕ^*_{jki}) for a firm from country i producing in country k and selling in country j. As in the previous cases we have two cases:

Case 5 If all the BMP cut-off productivities for firms from country i producing in country k (ϕ^*_{jki} $\forall j$) are above the MP cut-off productivity for firms from country i producing in country k (ϕ^*_{kki}), then the BMP cut-offs are well calculated. Firms with productivities $\phi^*_{kki} < \phi < \phi^*_{jki}$ sell domestically and produce and sell in country k, while firms with productivities $\phi > \phi^*_{jki}$ sell domestically, produce and sell in country k and also do BMP from country k to country j.

Case 6 If at least one BMP cut-off for firms from country i producing in country k (ϕ^*_{jki} $\forall j$) is below the MP cut-off productivity for firms from country i producing in country k (ϕ^*_{kki}), then we have to re-calculate the MP cut-off ϕ^*_{kki}. Define J^{BMP}_{ki} the set of countries for which the BMP cut-off (ϕ^*_{jki}) is lower than the MP cut-off (ϕ^*_{kki}). Then cut-off productivity for the marginal firm from country i performing MP in country k and BMP to country j solves:

$$\pi_{kki}(\phi^*_{kki}) + \sum_{j \in J^{BMP}_{ki}} \pi_{jki}(\phi^*_{kki}) = 0 \tag{1.14}$$

As firms performing BMP has to pay the fixed cost of producing abroad (κ^{MP}) also, there will be no firm performing BMP and not MP, which implies that the equilibrium BMP cut-off is not going to be below the MP cut-off. In Appendix 3 I present the algorithm to calculate the cut-offs.

Profits

In summary, if $\phi^*_{iii} < \phi^*_{kii}$, $\phi^*_{iii} < \phi^*_{kki}$ and $\phi^*_{kki} < \phi^*_{jki}$ all the cut-offs are the ones that come from equating the profit from each activity to zero, and so the marginal firm performing each activity makes zero profit. Otherwise the marginal firm entering in the domestic market can be making negative profits in the domestic market and compensate these negative profits with positive profits in other activities, like exporting or MP or both. Then, the profit made by a firm from country i is given by: to define profits I need to use an indicator that allows me to know if an activity is operative or not

$$\pi_i(\phi) = \pi_{iii}(\phi) + \sum_{k\neq i} \pi_{kii}(\phi) I^x_{kii} + \sum_{k\neq i} \pi_{kki}(\phi) I^{MP}_{kki} + \sum_{k\neq i}\sum_{j\neq k} \pi_{jki}(\phi) I^{BMP}_{jki} \ , \tag{1.15}$$

where I^x_{kii} is an indicator function that takes the value 1 if $\phi > \phi^*_{kii}$ and 0 otherwise, I^{MP}_{kki} is an indicator function that takes the value 1 if $\phi > \phi^*_{kki}$ and 0 otherwise, and finally I^{BMP}_{jki} is an indicator function that takes the value 1 if $\phi > \phi^*_{jki}$ and 0. [15] Note that for a firm with productivity ϕ it can be possible that the profit for some activities are negative. For example, it can happen that for this firm the profit of opening a plant in country k and selling to country k ($\pi_{kki(\phi})$, but the profit of producing in country k and selling to country j are positive and more than compensates the negative profit. Finally, as profits from every activity are increasing in ϕ (since $\sigma - 1 > 0$), more productive firms make higher profits, and so if the productivity is high enough a firm performs all the activities.

1.3.4.2 Productivity distribution

Productivities are drawn from a Pareto distribution with scale parameter ϕ^m_i and shape parameter α_i.[16] Lets define the density function as $g_i(\phi) = \alpha_i \frac{(\phi^m_i)^{\alpha_i}}{\phi^{\alpha_i+1}}$. As only firms with productivities above ϕ^*_{iii} will produce in country i, then the equilibrium distribution of productivities of domestic firms is:

$$\mu_i(\phi) = \frac{g_i(\phi)}{1 - G(\phi^*_{iii})} \text{ if } \phi \geq \phi^*_{iii} \ , \tag{1.16}$$

and 0 otherwise. The conditional probability of performing each of the other activities is:

$$\begin{aligned}
\text{Exporting to country } k \Rightarrow \theta_{kii} &= \frac{1 - G(\phi^*_{kii})}{1 - G(\phi^*_{iii})} \\
\text{Doing FDI in country } k \Rightarrow \theta_{kki} &= \frac{1 - G(\phi^*_{kki})}{1 - G(\phi^*_{iii})} \\
\text{Doing BMP in } k \text{ to sell in } j \Rightarrow \theta_{jki} &= \frac{1 - G(\phi^*_{jki})}{1 - G(\phi^*_{iii})}
\end{aligned}$$

The average productivity for each activity can be calculated as:

[15] In the calibrated model economies there are no exporting or MP cut-offs lower than the domestic cut-off. However, there are some BMP cut-offs smaller than the MP cut-offs.

[16] In a Pareto distribution the scale parameter indicates the minimum value that the random variable can take.

$$\tilde{\phi}_{jki} = \left[\int_{\phi^*_{jki}}^{\infty} \phi^{\sigma-1} \mu_i(\phi) d\phi \right]^{1/(\sigma-1)} \tag{1.17}$$

for all i, j and k. Notice that $\tilde{\phi}_{jki}$ only depends on the cut-off productivity.
Following Melitz (2003), we can consider that for each activity there is a representative firm with productivity $\tilde{\phi}_{jki}$. The average productivity $\tilde{\phi}_{jki}$ summarizes all the information concerning each activity. This is convenient because now aggregate variables for each activity can be expressed in terms of $\tilde{\phi}_{jki}$. One difference with respect to the case of Melitz (2003) is that in his case it is possible to calculate an average productivity for the whole economy that only depends on domestic firms. In this paper, the average productivity of a country will be given by the domestic firms producing domestically and also by foreign firms producing domestically. Then, aggregate variables for the whole economy will depend not only on the domestic mass of firms but also on the mass of firms from the rest of countries.

Evaluating revenues at the average productivity level and making the ratio of this revenue with a revenue evaluated at any other productivity level we find that:

$$\frac{r(\tilde{\phi}_{iii})}{r_{iii}(\phi)} = \frac{E_i^1 P_i^{\sigma-1} \left(\frac{\rho\tilde{\phi}_{iii}}{w_i}\right)^{\sigma-1}}{E_i^1 P_i^{\sigma-1} \left(\frac{\rho\phi}{w_i}\right)^{\sigma-1}} \Rightarrow r(\tilde{\phi_{iii}}) = \left(\frac{\tilde{\phi}_{iii}}{\phi}\right)^{\sigma-1} r_{iii}(\phi) \tag{1.18}$$

We can get the previous relation for each activity: exporting, doing MP and doing BMP.

$$\begin{aligned}
\text{Exporting to country } k \Rightarrow r(\tilde{\phi}_{kii}) &= \left(\frac{\tilde{\phi}_{kii}}{\phi}\right)^{\sigma-1} r_{kii}(\phi) \\
\text{Doing MP in country } k \Rightarrow r(\tilde{\phi}_{kki}) &= \left(\frac{\tilde{\phi}_{kki}}{\phi}\right)^{\sigma-1} r_{kki}(\phi) \\
\text{Doing BMP in } k \text{ to sell in } j \Rightarrow r(\tilde{\phi}_{jki}) &= \left(\frac{\tilde{\phi}_{jki}}{\phi}\right)^{\sigma-1} r_{jki}(\phi)
\end{aligned}$$

1.3.4.3 Sales distribution

Sales for a given activity are given by $r_{jki} = E_j^1 \left(\frac{P_j \rho \phi}{w_k \gamma_{ki} \tau_{jk}}\right)^{\sigma-1}$, where E_j^1 is aggregate expenditure in differentiated goods in country j. Given that productivities are drawn from a Pareto distribution it is possible to obtain the distribution of sales for each activity analytically. I will present the result for domestic firms selling domestically, but the expression is analog for the other activities.

$$\begin{aligned} prob(r_{iii}(\phi) > y) &= prob\left(E_i^1\left(\frac{P_i\rho\phi}{w_i}\right)^{\sigma-1} > y\right) \\ &= prob\left(\phi > \left(\frac{y}{E_i^1}\right)^{\frac{w_i}{1-\sigma}} \frac{w_i}{P_i\rho}\right) . \end{aligned}$$

As ϕ is distributed Pareto we can calculate this probability to be

$$prob(r_{iii}(\phi) > y) = \left(\frac{\phi_i^m}{\left(\frac{y}{E_i^1}\right)^{\frac{1}{1-\sigma}} \frac{w_i}{P_i\rho}}\right)^{\alpha} ,$$

where $\phi_{m,i}$ is the scale parameter (the minimum value that ϕ can take) of the Pareto distribution. We can write the above expression as:

$$\begin{aligned} prob(r_{iii}(\phi) > y) &= \left(\frac{(E_i^1)^{1/(\sigma-1)}(P_i\rho\phi_i^m/w_i)}{y^{1/(\sigma-1)}}\right)^{\alpha} \\ prob(r_{iii}(\phi) > y) &= \left(\frac{E_i^1(P_i\rho\phi_i^m/w_i)^{(\sigma-1)}}{y}\right)^{\alpha/(\sigma-1)} \\ prob(r_{iii}(\phi) > y) &= \left(\frac{r_i^m}{y}\right)^{\alpha/(\sigma-1)} \end{aligned}$$

where $r_i^m(\phi_i^m) = E_i^1(P_i\rho\phi_i^m)^{\sigma-1}$ is the revenue of a firm from country i with productivity equal to $\phi_{m,i}$ producing and selling domestically. Then $r_{iii}(\phi)$ is distributed Pareto with scale parameter $r_{m,i}$ and shape parameter $\alpha/(\sigma-1)$. This would be the distribution of sales if all the firms were producing. But, as we stated previously, there will be some firms (the ones with productivity between $\phi_{m,i}$ and ϕ_{iii}^*) which are not going to produce. Then, the true distribution of sales will be a truncation of the previous distribution. The Pareto distribution has the property that if it is truncated, the remaining distribution is still Pareto with the same shape parameter. Then sales $(r_{iii}(\phi))$ are distributed Pareto with scale parameter $r_{iii}(\phi^*)$ and shape parameter $\alpha/(\sigma-1)$, where $r_{iii}(\phi^*)$ are the sales of a firm with the cut-off productivity.

For the rest of activities we can operate in a similar way and we obtain:

$$\begin{aligned}
\text{Exporting firms} \Rightarrow prob(r_{kii} > y) &= \left(\frac{E_k^1 \left(\frac{P_k \rho \phi_i^m}{w_k \tau_{ki}} \right)^{(\sigma-1)}}{y} \right)^{\alpha/(\sigma-1)} \\
\text{Doing FDI in country } k \Rightarrow prob(r_{kki} > y) &= \left(\frac{E_k^1 \left(\frac{P_k \rho \phi_i^m}{w_k \gamma_{ki}} \right)^{(\sigma-1)}}{y} \right)^{\alpha/(\sigma-1)} \\
\text{Doing BMP in } k \text{ to sell in } j \Rightarrow prob(r_{jki} > y) &= \left(\frac{E_j^1 \left(\frac{P_j \rho \phi_i^m}{w_k \tau_{jk} \gamma_{ki}} \right)^{(\sigma-1)}}{y} \right)^{\alpha/(\sigma-1)}
\end{aligned}$$

where the numerator of each equation are the sales for each activity that correspond to the minimum productivity level. As in the case of domestic sales, the equilibrium distribution of sales for each activity is going to be Pareto with shape parameter $\alpha/(\sigma-1)$ and scale parameter $r(\phi^*_{jki})$, where $r(\phi^*_{jki})$ is sales of a firm with the cut-off productivity level for a firm from country i producing in country k and selling to country j.

1.3.4.4 Average Profits

Replacing (1.17) in the profit equations we can calculate average profits in terms of average productivities. In the case that each individual activity makes zero profit at the cut-off level, we can calculate average profit for each activity as:

$$\begin{aligned}
\text{Selling Domestically} \Rightarrow \bar{\pi}_{iii} &= \kappa_i^d w_i \left[\left(\frac{\tilde{\phi}_{iii}}{\phi^*_{iii}} \right)^{\sigma-1} - 1 \right] \\
\text{Exporting from the home country} \Rightarrow \bar{\pi}_{kii} &= \kappa_i^x w_i \left[\left(\frac{\tilde{\phi}_{kii}}{\phi^*_{kii}} \right)^{\sigma-1} - 1 \right] \\
\text{Doing MP in country } k \Rightarrow \bar{\pi}_{kki} &= \kappa_k^{MP} w_k \left[\left(\frac{\tilde{\phi}_{kki}}{\phi^*_{kki}} \right)^{\sigma-1} - 1 \right] \\
\text{Doing BMP in } k \text{ to sell in } j \Rightarrow \bar{\pi}_{jki} &= \kappa_k^x w_k \left[\left(\frac{\tilde{\phi}_{jki}}{\phi^*_{jki}} \right)^{\sigma-1} - 1 \right]
\end{aligned}$$

If the profit at the cut-off level is not zero, then the average profit for that activity is obtained using equation (1.18). We can calculate the average profit of a firm from country i as:

$$\bar{\pi}_i = \bar{\pi}_{iii} + \sum_{k \neq i} \theta_{kii} \bar{\pi}_{kii} + \sum_{k \neq i} \theta_{kki} \bar{\pi}_{kki} + \sum_{k \neq j} \sum_{k \neq i} \theta_{jki} \bar{\pi}_{jki} \ . \qquad (1.19)$$

Notice that profits are a function of aggregate expenditures E_i^1. Aggregate expenditure is determined, among other factors, by the population size. Hence, the profitability of a foreign firm depends on the selling country size. Given two countries with similar variable and fixed trade costs, a multinational plant will prefer to get installed in the bigger country. As a result, a small country will attract less investment than a bigger one. For example, assume that the country where the good is going to be consumed is Uruguay, and a firm from Japan is considering the different possibilities of serving Uruguay. If the fixed export cost in Japan is high, then it could be better to produce the good directly in Uruguay. This will be the case if the fixed cost to open a subsidiary in Uruguay is not very high and also the productivity loss for producing abroad ($\gamma_{Uru,Jap}$) is low. Now, imagine that Japan is also considering to sell to Argentina, and that the productivity loss of producing in Argentina for a Japanese firm is the same as in Uruguay $\gamma_{Arg,Jap} = \gamma_{Uru,Jap}$. Then, as Argentina is bigger, $E_{Arg}^1 > E_{Uru}^1$. If aggregate prices, wages, and fixed costs are not very different, the Japanese firm will prefer to produce in Argentina to producing in Uruguay. In other words, the productivity required by a Japanese firm to start producing in Argentina is lower (ceteris paribus) than the one required to produce in Uruguay. This implies that more firms get located in Argentina. Size, then, is crucial to attract foreign investment.

1.3.4.5 Mass of Firms

Define M_i^e to be the total mass of firms making a productivity draw in country i, and M_i as the mass of firms finally operating. By definition, the total mass of firms operating should be equal to the total mass of firms making a productivity draw times the probability of successful entry, which is $\theta_{iii} M_i^e = M_i$.

In the case of an open economy without FDI we can obtain M_i in the same way as in Melitz (2003). $M_i = R_i/\bar{r}_i$, where $R_i = w_i L_i$ denotes aggregate revenue and aggregate expenditure, and $\bar{r}_i$ denotes average revenue. In Melitz (2003), aggregate revenue and total payment to labor are equal because total profits (Π) are equal to the payment to labor used in making the productivity draw ($\kappa_i^e w_i$) in equilibrium and only domestic firms produce in country i.

When foreign firms are allowed to produce in the domestic country $R_i \neq w_i L_i$. The equality does not hold because foreign firms send their profits abroad, and domestic firms producing abroad bring their profits home, making total expenditure in the country also a function of profits of domestic firms abroad. However, it is still true that $w_i L_i^e = \Pi_i$ (where L_i^e is labor used in entering) [17], but the determination for labor used in production (L_i^p) is different. Now the total payment to labor in country i is equal to revenue minus profits of firms producing in

[17] This is obtained using the equation for total payment to labor used in entering and the free entry condition, which I explain later.

i, which can include foreign firms. In equations, $w_i L_i^p = \hat{R}_i - \hat{\Pi}_i$ where $\hat{R}_i$ and $\hat{\Pi}_i$ are revenues and profits of firms producing in country i (domestic or foreign).

The total mass of firms performing each of the other activities is obtained by multiplying the mass of firms operating, M_i, by the conditional probability of performing the activity $M_{jki} = \theta_{jki} M_i$.

1.3.4.6 Aggregation

We define aggregate price and GDP in country i as:

$$\begin{aligned} P_i &= \left[\int_{\phi^*_{iii}} (p_{iii}(\phi))^{1-\sigma} M_i \mu_i(\phi) d\phi + \sum_{k \neq i} \int_{\phi^*_{ikk}} (p_{ikk}(\phi))^{1-\sigma} M_k \mu_k(\phi) d\phi \right. \qquad (1.20) \\ &+ \left. \sum_{k \neq i} \int_{\phi^*_{iik}} (p_{iik}(\phi))^{1-\sigma} M_k \mu_k(\phi) d\phi + \sum_{k \neq j} \sum_{k \neq i} \int_{\phi^*_{ikj}} (p_{ikj}(\phi))^{1-\sigma} M_j \mu_j(\phi) d\phi \right]^{\frac{1}{1-\sigma}} , \end{aligned}$$

$$\begin{aligned} GDP_i &= \int_{\phi^*_{iii}} r_{iii}(\phi) M_i \mu_i d\phi + \sum_{k \neq i} \int_{\phi^*_{kii}} r_{kii}(\phi) M_i \mu_i d\phi + \sum_{k \neq i} \int_{\phi^*_{iik}} r_{iik}(\phi) M_k \mu_k d\phi \\ &+ \sum_{k \neq j} \sum_{k \neq i} \int_{\phi^*_{kij}} r_{kij}(\phi) M_j \mu_j d\phi \ . \qquad (1.21) \end{aligned}$$

We can re-write the aggregate price and GDP of country i in terms of weighted average productivities.[18] Lets define M_i^p as the mass of firms producing in country and M_i^s as the mass of firms selling goods to country i. Then,

$$\begin{aligned} M_i^p &= M_i + \sum_{k \neq i} M_{iik} + \sum_{k \neq i} \sum_{i \neq j} M_{jik} \ , \\ M_i^s &= M_i + \sum_{k \neq i} M_{iik} + \sum_{k \neq j} \sum_{i \neq j} M_{ijk} \ . \qquad (1.22) \end{aligned}$$

[18] Following Melitz (2003).

Having defined the mass of firms producing and selling in each country we can define the weighted average productivity of firms producing ($\tilde{\phi}_i^p$) and selling ($\tilde{\phi}_i^s$) as:

$$\begin{aligned}
\tilde{\phi}_i^p &= \left\{ \frac{1}{M_i^p} \left[M_{iii}\tilde{\phi}_{iii}^{\sigma-1} + \sum_{k\neq i} M_{kii} \frac{E_k^1}{E_i^1} \left(\frac{P_k}{\tau_{ki}P_i} \right)^{\sigma-1} \tilde{\phi}_{kii}^{\sigma-1} + \sum_{k\neq i} M_{iik} \left(\frac{1}{\gamma_{ik}} \right)^{\sigma-1} \tilde{\phi}_{iik}^{\sigma-1} \right.\right. \\
&+ \left.\left. \sum_{k\neq i} \sum_{i\neq j} M_{jik} \frac{E_j^1}{E_i^1} \left(\frac{P_j}{\tau_{ji}\gamma_{ik}} \right)^{\sigma-1} \tilde{\phi}_{jik}^{\sigma-1} \right] \right\}^{\frac{1}{\sigma-1}} , \qquad (1.23) \\
\tilde{\phi}_i^s &= \left\{ \frac{1}{M_i^s} \left[M_{iii}\tilde{\phi}_{iii}^{\sigma-1} + \sum_{k\neq i} M_{ikk} \left(\frac{w_k \tau_{ik}}{w_i} \right)^{1-\sigma} \tilde{\phi}_{ikk}^{\sigma-1} + \sum_{k\neq i} M_{iik} \gamma_{ik}^{1-\sigma} \tilde{\phi}_{iik}^{\sigma-1} \right.\right. \\
&+ \left.\left. \sum_{k\neq i} \sum_{i\neq j} M_{ijk} \left(\frac{\tau_{ij}\gamma_{jk}w_k}{w_i} \right)^{1-\sigma} \tilde{\phi}_{ijk}^{\sigma-1} \right] \right\}^{\frac{1}{\sigma-1}} . \qquad (1.24)
\end{aligned}$$

Using these two equations we can define aggregate price and aggregate production in the differentiated good sector in country i as:[19]

$$\begin{aligned}
P_i &= (M_i^s)^{\frac{1}{1-\sigma}} p(\tilde{\phi}_i^s) = (M_i^s)^{\frac{1}{1-\sigma}} \frac{w_i}{\rho \tilde{\phi}_i^s} , \qquad (1.25) \\
GDP_i &= M_i^p E_i \left(\frac{P_i \rho \tilde{\phi}_i^p}{w_i} \right)^{\sigma-1} . \qquad (1.26)
\end{aligned}$$

1.3.5 Trade and Multinational Production

Trade of a country will be given by the amount of exports and imports. Exports are composed by all the sales to foreign countries from firms (either domestic or foreign) producing in the domestic country. The expression for total exports in the differentiated good sector is given by:

$$\text{Exports}_i = X_i = \underbrace{\sum_{k\neq i} M_{kii} r_{kii}(\tilde{\phi}_{kii})}_{\text{Exports by Domestic Firms}} + \underbrace{\sum_{k\neq i} \sum_{k\neq j} M_{jik} r_{jik}(\tilde{\phi}_{jik})}_{\text{Exports by Foreign Firms}} .$$

In a similar way, imports in the differentiated good sector are all the goods consumed in the domestic country and produced in a foreign country. So total imports are:

$$\text{Imports}_i = IM_i = \sum_{k\neq i} M_{ikk} r_{ikk}(\tilde{\phi}_{ikk}) + \sum_{k\neq i} \sum_{k\neq j} M_{ijk} r_{ijk}(\tilde{\phi}_{ijk}) .$$

[19] Proof in the appendix

The capital account is composed by the difference between the profits of domestic firms producing abroad and the profits of foreign firms producing in the domestic country.

$$\text{Capital Account}_i = \sum_k \sum_{j\neq i} M_{kji}\bar{\pi}_{kji} - \sum_k \sum_{j\neq i} Mkij\bar{\pi}_{kij} \ .$$

The Current Account (CA) is the sum of Trade Balance (TB) $TB_i = (z_i - q_{i0}) + X_i - IM_i$ where $(z_i - q_{i0})$ is net exports of the homogeneous good, and the capital account balance. The current account balance equation can be written as:

$$\text{CA}_i = (z_i - q_{i0}) + X_i - IM_i + \sum_k \sum_{j\neq i} M_{kji}\bar{\pi}_{kji} - \sum_k \sum_{j\neq i} Mkij\bar{\pi}_{kij} \ . \tag{1.27}$$

1.3.6 Equilibrium

Equation (1.19) defines the Zero Cut-off Profit Condition (ZCPC), which expresses the average profit of a firm from country i as a function of the domestic cut-offs, the mass of firms operating in each country, and wages. The net value of a firm from country i is then $v_i = \bar{\pi}_i$. As there is free entry, the expected profit of a firm before making a draw should be zero, otherwise more firms will enter until this condition is satisfied. Define the net value of an entering firm as v_i^e. In equilibrium v_i^e should be equal to zero. Then the free entry condition (FEC) can be expressed as:

$$v_i^e = \theta_{iii}\bar{\pi}_i - \kappa_i^e w_i = 0 \ , \tag{1.28}$$

which says that the average value of a firm producing in country i times the probability of successful entry should be equal to the entry cost (the cost of making the productivity draw). θ_{iii} is a function of the scale (ϕ_i^m) and shape (α) parameters of the productivity distribution and of the domestic cut-off (ϕ_{iii}^*). Rearranging terms in equation (1.28) we get $\bar{\pi}_i = \frac{\kappa_i^e w_i}{\theta_{iii}}$.

In order to solve for the equilibrium we need to find $3*N+1$ variables: N cut-offs ($\phi_{iii}\ \forall i$); N numbers for the mass of firms for each country ($M_i\ \forall i$); N wages (w_i) and 1 price (p_0) . Normalizing the price of the homogeneous good to one we end up with $3*N$ endogenous variables. The set of $3*N equations$ are given by:

- Free entry condition equal to zero cut-off profit condition.
- Current account balance condition.
- Labor market clearing condition.

Definition: *Given z_{i0}, τ_{ij}, γ_{ij}, κ_i^e, κ_i^d, κ_i^x, κ_i^{MP}, $g_i(\phi)$, L_i and N $\forall i,j=1,...,N$, a* ***multinational production equilibrium*** *is a set of wages w_i, price indices, P_i, income, GNP_i,mass of* *firms M_i, mass of entrants, M_i^e, allocations for the representative consumer $q_{jki}(\phi)$ and prices, $p_{jki}(\phi)$, for firms such that:*

1. *In all countries, given prices and aggregate expenditure, consumers demand choices ($q_{jki}(\phi)$ and q_{io}) satisfy* (1.2) *and* (1.3)*.*
2. *In all countries, firms maximize profits from all activities (equation* (1.5) *solves* (1.4)*).*
3. *P_i satisfies equation* (1.20)
4. *Labor markets clear.*
5. *Free entry condition: $v_i^e = 0$ (see equation* (1.28)*).*
6. *Current Account balance condition is zero (see equation* (1.27)*).*
7. *The mass of firms producing is equal to the mass of firms taking the productivity draw times the probability that the draw is bigger than the domestic cut-off,* $M_i = \theta_{iii} M_i^e$
8. *World demand of the homogeneous good is equal to world supply: $\sum_i z_i = \sum_i q_{i0}$.*

1.4 Calibration

1.4.1 Data

I use data from four different sources to calibrate the model: The World Bank Enterprise Survey (WBES), the United Nations (UNCTAD), OECD Stan, and the database on bilateral trade flows from Waugh (2010).

World Bank Enterprise Survey: This database is a stratified sample of the universe of firms in developing countries. I use the standardized survey, which has data starting in 2006. This database is being updated continuously, and for many countries there is a panel of two years already. I use this database to obtain statistics related to firms' performance for South American countries : a) proportion of exporting firms; b) proportion of foreign firms. I consider only firms in the manufacturing sector.

UNCTAD: I use the Foreign Direct Investment profile for the Latin-American countries under study. I use data on the origin of the stock of FDI by country.

OECD Stan: I use data on the production by multinational firms and proportion of firms exporting for Europe.

Waugh(2010): This data base contains information on trade for a large set of countries for the year 1996, including Latin-American and European countries. I use trade statistics (exports and imports) by origin and destiny in order to construct bilateral trade flows between countries and the absorption measure reported.

1.4.2 Calibration Strategy:

I calibrate the model separately for two regions: South America and Europe. I select these regions because they both present very different trade arrangements. While South America is characterized by high trade barriers, Europe is well known as a low trade barriers' area for the members of the European Union. Analyzing the differences in the gains from openness in these two regions for countries of different size provides information on how much countries in the close region are loosing compare to the ones in the open region, and how much could be the potential gains of becoming more open. To maintain symmetry I will include in both calibration five countries, four belonging to the region and a fifth which stands for the rest of the world (RW). The countries included in each regions are (i) Argentina, Brazil, Chile, Uruguay in South America; and (ii)France, Italy, Netherlands and United Kingdom in Europe.

I will use data for 1996 whenever it is possible.[20] OECD Stan database has information on sales of multinationals only for the late 2000's. I will use data for 2007 which is the earliest year for which they have data for all countries. The parameters I need to calibrate are:

- **Size** (L_i): I use data from the UNCTAD on labor force. I normalize Uruguay's size to 1 ($L_{Uru} = 1$). Countries sizes are then $L_{Arg} = 9.47$, $L_{Bra} = 48.94$, $L_{Chi} = 3.69$ and $L_{RW} = 1582.5$.[21] For Europe country sizes are: $L_{Fra} = 16.8$, $L_{Ita} = 14.9$, $L_{UK} = 18.7$ and the $L_{RW} = 1567.3$.

- **Substitutability between varieties (σ):** I use a value of 6 which generates a mark-up of 20%, as is common in the literature (for example in Ghironi and Melitz (2005)).

- **Productivity distribution:** I assume that productivities are drawn from a Pareto distribution with scale parameter $\phi_m = 1$ for all countries. I will assume that all countries have the same shape parameter α. Given the Pareto assumption for productivities, sales are distributed Pareto with shape parameter $\alpha/(\sigma - 1)$. There is a large discussion in the literature about the value of α and $\alpha/(\sigma-1)$. Chaney (2008) finds that $\alpha_i/(\sigma-1)$ is around 2 for the US, but he does not calculate the value of α and σ. Ramondo and Rappoport (2010) use $\alpha = 4$. Breinlich and Cuñat (2010) estimates $\alpha/(\sigma - 1)$ and find values ranging

[20] In 1995 the MERCOSUR members should have had the last reduction in tariffs for trade within the region, and a common tariff for the rest of the world. For a more detailed discussion on this see Bustos (2011).

[21] To calculate the RW I take out Russia and Germany from all the variables, two big countries not included in Waugh (2010).

from 1.13 to 4.88. Arkolakis, Ramondo, Rodriguez-Clare, and Yeaple (2013) use $\alpha = 4.2$. Finally, Arkolakis and Muendler (2010) estimates $\alpha/(\sigma-1)$ from Brazilian data and find a value of 1.21. I use the estimate of Arkolakis and Muendler (2010) for two reasons. First, because they estimate the shape parameter of sales from Brazilian data, one of the countries I am studying. Second, because $\sigma = 6$ implies $\alpha = 6.05$ which is in the middle range of previous estimates.

- **Fixed entry cost (κ_i^e):** In order to make a productivity draw, firms in country i should pay a fixed cost κ_i^e. I calibrate this parameter to match the GDP per capita in each country relative to the RW for the year 1996.

- **Fixed operating cost (κ_i^d):** If a firm decides to operate, it has to pay a fixed cost (κ_i^d). I will set the value of this parameter such that the smallest firm producing in each country demands 10 workers. The amount of labor demanded by the smallest firm is:

$$\ell(\phi_{iii}^*) = \sigma\kappa_i^d \ . \tag{1.29}$$

As equation (1.29) shows, labor demand of the firm with productivity level equal to the domestic cut-off productivity only depends on σ and κ_i^d.[22] As all countries have the same σ, all countries should have the same κ_i^d in order to obtain that the smallest firm demands ten workers in all countries. I thus set $\kappa_i^d = 10/6$ for all i.

- **Fixed cost of exporting (κ_i^x):** In order to export, a firm has to pay an additional fixed cost (κ_i^x). This cost directly affects the mass of firms deciding to export. I will use the proportion of firms exporting as a fraction of the total number of operating firms. For South America I use firm-level data from the World Bank Enterprise Survey to calculate this statistic in the data. For Europe, I use the OECD Stan dataset.

- **Fixed cost of doing MP (κ_i^{MP}):** To operate in a foreign country, a firm has to pay a fixed cost of (κ_i^{MP}) in the country where the firm will open the plant. I will calibrate this parameter to match the proportion of foreign firms in a given country. As this cost increases, the proportion of foreign firms decreases. I use data from the World Bank Enterprise Survey to construct this statistic in the data for South America and OECD Stan for Europe.

- **Iceberg cost of exporting (τ_{ji}):** In order to deliver one unit to country j, firm in country i has to deliver τ_{ji} units. These parameters are pinned down to target $Trade_{ji}$ over $Absorption_i$[23] across the countries in my study. I use data from Waugh (2010) on trade of manufactures to construct these targets.

[22] See proof in the appendix.

[23] $Trade_{ji}$ is imports of country i from country j plus exports from country i to country j. Absorption is calculated as $GDP_i + Imports_i - Exports_i$.

- **Productivity shifter (γ_{ji}):** When a firm produces abroad the productivity of a firm is shift by γ_{ji}. The new productivity for a firm from country i producing in country j is $\hat{\phi} = \frac{\phi}{\gamma_{ji}}$ To calibrate this parameter I use the proportion of sales from foreign firms in the domestic country. I do not allow firms from the countries in the sample to perform FDI in the rest of the world. Using data from the WBES I compute the participation of foreign sales on total sales. Unfortunately, this database does not have the country of origin of foreign firms. So, I use the composition of FDI stock in manufactures to impute these values. The data on FDI stock in manufactures come from the UNCTAD Foreign Direct Investment profile for South America and OECD Stan for Europe.

- **Endowment of the homogeneous good (z_i):** I use the trade deficit in the manufacturing sector to calibrate this parameter.

1.4.3 Calibration Results

Tables 1.1 to 1.6 present the calibrated parameters. Panel A of each table presents the results for South America, while Panel B presents the results for Europe. The model performs well in matching the selected targets. The GDP per capita of the RW is normalized to 1. To match the much higher GDP per capita in Europe relative to the RW (see Table 1.6), I need to impose much lower entry costs in Europe than in South America (second column of Table 1.1). Table 1.2 shows that the model also matches the trade balance over absorption in the manufacturing sector, even though it slightly overestimates Italian trade surplus (9.5 in the model versus 8.9 in data). For the proportion of firms exporting (first column of Table 1.1) and the participation of foreign firms sales in total sales (Table 1.5), the model is able to match the data almost perfectly for all countries.

To match the trade statistics I use variable and fixed trade costs. Note that Argentina and Brazil, the two largest countries in South America, show lower ratios of Trade-to-Absorption, 35.8% and 22.8% respectively. On the other hand, Chile and Uruguay, the smallest countries, show much higher ratios: 59.4% and 58.3%. In order to match the large proportion of domestic firms exporting in Argentina, the model requires small fixed cost of exporting for this country (see column four of Table 1.1). This also allows smaller firms to enter in the export market, making it possible to match at the same time the large proportion of firms exporting and the relatively low trade-to-absorption ratio. The importance of the RW as a trade partner is also shown in the calibrated parameters. Participation of the RW in trade goes from 51% for Uruguay to 86% for Chile. As a result Chile, with high variable trade cost of exporting to the rest of South American countries, presents low average variable trade cost (compared to the levels of Argentina and Brazil around 100%). Uruguay is the country of the region with the highest average variable trade costs (124%), something unexpected since it is the smallest country.[24] For Europe we can immediately observe that trade-to-absorption is much higher than in South America. Italy, the country with the lowest ratio has a value of 44.1%, while the Netherlands, the country with the highest ratio, exhibits a ratio of 118.1%. In order to match the higher ratio, the model requires much smaller trade costs. This is shown in Figure 1.3, which shows the average trade costs by country (both the simple average or a weighted by trade composition average). It can be easily seen that South American countries face much higher average trade costs than European countries. The weighted average trade cost in South America is 111% (so the average variable cost is $\tau = 2.11$), while in Europe it is 65% (the average variable cost in Europe is $\tau = 1.649$). Another interesting fact is that while in Europe the smallest country, the Netherlands, faces the lowest trade cost, in South America the smallest country, Uruguay, faces the highest average trade costs.

Similar observations apply to multinational production.[25] As in the case of trade costs, the efficiency parameter γ is much higher for South America than Europe (see Table 1.5). This implies that foreign firms are much less productive operating abroad in South America than in Europe. The average value of this parameter is 1.92 in South America, while is 0.58 in Europe. The fact that in Europe the average γ is smaller than one is mainly driven by the productivity of firms from the RW operating abroad. Firms from the RW operating in Europe are three times more efficient than in their domestic countries. Then, as most of the MP comes from the RW, the average γ in Europe is smaller than one.

To sum up, the baseline model is consistent with cross country evidence on bilateral trade flows and multinational production for the set of selected countries. South America faces higher trade barriers than Europe, and this trade barriers vary with country size among regions. While the smallest country in Europe (the Netherlands) is the one with the smallest average trade costs, Uruguay, the smallest country in south America is the one with the highest average trade cost. Also, South American countries can not attract as much MP as European countries because the productivity of multinationals operating in South America is much lower than the productivity of multinationals operating in Europe.

1.5 Experiments

I use the calibrated model to perform a set of counterfactual experiments. First, I investigate how much countries benefit from trade and MP by closing the economies (a world in autarky), and study the role played by MP and BMP in countries of different size. Then, I reduce trade and MP costs in South America and study the potential gains in real GDP. Finally, I analyze

[24] Small countries are those who benefit the most from openness according to traditional trade theory

[25] I set a value of 100 to γ_{ij} when there are zeros in the data

the role of trade and MP in shaping the distribution of firm size in countries of different size in the two regions. In summary, I will quantitative study the following:

1. To assess the low gains from trade and MP attained by South America relative to Europe, I compute the losses (changes in real manufacturing GDP and GNP) of moving to autarky in South America relative to Europe.
2. To assess the role played by MP and BMP in explaining the previous results I perform three exercises:
   - To assess the role played by BMP, I compare the losses of moving to autarky in a world with and without BMP.
   - To assess the role of MP itself (without including BMP), I set up a world without BMP, and compare the losses of moving to autarky with and without MP.
   - To assess the role played by MP as a whole (including BMP), I compare the losses of moving to autarky in a world with and without MP.
3. To assess the potential gains from an improvement in the degree of openness in South America, I compute the changes in real manufacturing GDP and GNP of decreasing trade costs in South America to the average level in Europe with three different configuration:
   - Maintaining the same multinational production costs.
   - Increasing the efficiency of foreign firms producing in South America by 20%.
   - Increasing the efficiency of foreign firms producing only in Uruguay by 20%.
4. To assess the effects of MP and trade on firm size distribution, I compute the proportion of firms with more than 100 and 250 employees in the baseline economy, in an economy without MP and in an economy in autarky.

1.5.1 Gains from Openness

To study the gains from openness, I close the economies to trade and MP (a world in autarky). In autarky, $\gamma_{ij} = \tau_{ij} = \infty$. The first two columns of Table 1.7 present the changes in real manufacturing GDP and GNP using as benchmark the calibrated model economies. Panel A presents the results for South America and Panel B for Europe. Losses of moving to autarky in Europe are much larger than in South America (10.5% versus 5.3% of real GDP) which indicates that Europe benefits much more from openness than South America. This is expected since trade costs are higher and efficiency of foreign firms is lower in South America compared to Europe. Small countries lose more than large countries in both regions. The higher degree of openness in Europe results in larger differences between the country that loses the most and the country that loses the least compared to South America. In Europe, the Netherlands loses 20.3% of real GDP and Italy loses 5.6% (almost 15 percentage points difference), while in South America, the difference between the losses of Uruguay and Brazil is 8.5 percentage points.

The last two columns of Table 1.7 present the changes in real manufacturing GDP and GNP using as benchmark a modified version of the baseline economy, an economy where BMP is not allowed. Allowing for BMP introduces an extra possibility for foreign firms, the possibility of using a third country as an export platform. However, the extent to which they will be able to benefit from exports will be determined by trade barriers. Comparing the results of the third column, to the one of the first column, we can see that BMP is more important in small countries than in large countries, and that European countries benefit more from BMP. The losses for the Netherlands in a world without BMP are 3.3 p.p lower than in the benchmark economy, while for Uruguay are only 1.2 p.p. lower. Then, high trade barriers not only affect exports of domestic firms but also exports of foreign firms, and as a result the ability of small countries to attract multinational firms.

1.5.1.1 The role played by MP and BMP

To disentangle the role played by MP and BMP in the gains from openness, I perform three experiments. The results of these three exercises are presented in Table 1.8.

To assess the role played by BMP in explaining the gains from openness, I compute the ratio between the losses of going to autarky in the baseline economy and the losses of going to autarky in a world without BMP. If this ratio is 1 it means that BMP plays no role in explaining the gains from openness, while BMP becomes more important as this ratio increases. The first column of Table 1.8 presents the results. The ratio is larger for small countries than for large countries, and also tends to be higher in Europe than in South America. In Uruguay the losses of going to autarky in the baseline economy are 11.2% higher than in the case without BMP and in the Netherlands this number goes up to 19.9%, while for Brazil and Italy this ratio is 1.035 and 1.097 respectively. Then, small countries benefit more from BMP as expected.

To assess the role played by MP itself (this means MP without the possibility of BMP), I compute the ratio between the losses of going to autarky in a world without BMP and the losses of going to autarky in a world without MP. This ratio shows the importance of MP itself in explaining the gains from MP as a whole. If the ratio is close to 1 it means that the gains from MP mostly come through BMP, while as this ratio increases it means that MP itself becomes more important (and as a result BMP becomes less important) in explaining the gains from MP. The second column of Table 1.8 presents the results. In Europe, Italy loses 84% more going to autarky in a world without MP compared to a world without BMP, but the Netherlands only loses 20% more. As expected MP itself is more important in explaining the gains from MP in the large country than in the small one. In South America both Brazil and Uruguay present the same ratio 1.51. This happens because the efficiency of multinationals firms operating in Brazil is very low, and so MP is not a very cheap way of overcoming trade barriers. However, another large country like Argentina has a ratio much higher (2.033) than the one of Uruguay.

Finally, to asses the role played by MP through both channels, I compute the ratio between the losses of going to autarky in the baseline economy and the losses of going to autarky in a world without MP. If this ratio is 1 it means that MP plays no role in explaining the gains from openness, while as it increases it means that MP becomes more important in explaining the gains from openness. The third column of Table 1.8 presents the results. Large countries display the highest gains from MP. In South America the country that benefits the most is Argentina and in Europe Italy. In Europe, the country that benefit the least from MP is the Netherlands, the smallest country, while in South America is Brazil the largest country. The underlying message is the same as in the previous exercise: since in Brazil the efficiency of multinational firms is low, the role played by MP is lower. Also, as South America as a region is closed then the gains from trade are not very large which increases the importance of MP in explaining the gains from openness.

To sum up, if countries face relatively low trade costs and high efficiency of foreign firms, large countries benefit more from MP as a whole, with small countries benefitting more from BMP. On the other hand, if trade costs are high and the efficiency of multinationals is low the large country may not benefit from MP more than the small country.

1.5.2 Reducing trade costs and improving efficiency

To study the potential gains in South America of an improvement in the degree of openness, I reduce the average trade costs for all countries in the calibration for South America to the average level in Europe ($\tau = 1.64$).[26]

Panel A of Table 1.9 presents the result of only reducing trade costs in South America to the average level in Europe (i.e. imposing $\tau = 1.64$ to all South American countries). All countries gain by reducing trade costs, but the smallest country, Uruguay, gains significantly more. The gains in Uruguay are 29.9% of real manufacturing GDP, while in Brazil, the largest country, are just 4%. I find, as Eaton and Kortum (2002), that the gains from reducing trade costs are larger than the losses of going to autarky. Thus, just by reducing trade barriers South America can obtain large gains.

To assess the potential gains South American countries may obtain from the interaction of trade and MP, in addition to the reduction in trade costs I increase the productivity of multinational

[26]Trade barriers can be reduced by reducing trade tariffs within the region and also with the RW, improving the available infrastructures, forcing countries to respect trade agreements, etc.

firms by 20%. Panel B of Table 1.9 presents the results of this experiment. There is a large gain in real manufacturing GDP in all countries, but specially in large countries. However, since multinational firms send their profits back, the increase is not reflected in a large increase in real manufacturing GNP, except for Uruguay. In Uruguay, real manufacturing GDP increases more than 9 percentage points relative to the previous experiment, while real manufacturing GNP increases almost 7 percentage points more relative to the previous experiment.

Panel C of Table 1.9 presents the result of only increasing the efficiency to multinationals operating in Uruguay by the same magnitude as in the previous exercise. Changes in real manufacturing GDP for the rest of countries are the same as in the case of only reducing trade costs, while in Uruguay it increases by 12 percentage points in addition. An interesting result from these experiments is that Uruguay would gain more if the efficiency improves only domestically compared to the case where it improves in all the countries of the region. This is because if the efficiency only improves in Uruguay there is a larger set of multinationals going to this country.

Discussion on Bridge Multinational Production

The previous experiments reflect the importance of BMP for a small country. In the absence of BMP, the gains in real manufacturing GDP of reducing trade barriers decrease for all countries, but they decrease significantly more for Uruguay. In Uruguay the gains are reduced by 6.2 p.p. while in Brazil they are only reduced in 0.3 p.p. (see Panel A of Table 1.9, column 3). When trade costs are reduced, small countries can attract more foreign firms who will locate there to export to the rest of countries, explaining the importance of BMP. This indicates that, for small countries like Uruguay to take advantage of MP, it needs to be able to export to the rest of countries in the region.

Panel B of Table 1.9 shows in the third column the increase in manufacturing real GDP when in addition to the reduction in trade costs we increase efficiency of foreign firms but we do not allow for BMP. Compared to the numbers in Panel A we see that Uruguay is the country with the smallest additional increase in manufacturing real GDP (0.7 p.p.) while the rest of countries show increases that goes from 1.9 p.p. to 4 p.p. This result indicates that BMP is crucial for Uruguay to benefit from increases in efficiency of multinationals since otherwise the gains would not be larger than the ones it would get by only reducing trade costs.

Finally, if we only improved the efficiency of foreign firms operating in Uruguay we again find that BMP is crucial to explain the gains. While in the baseline economy real manufacturing GDP increases 10.9 p.p. more than when we only reduce trade costs, if we shut down BMP the additional increase is only of 2.3 p.p. The result is explained because without the possibility of serving third countries Uruguay does not become an attractive location for multinational firms, even with the increase in productivity, because its domestic market is small.

Discussion on the Role of Assumption 1

Assumption 1 allows me to treat each activity as independent. With assumption 1 a firm located in Uruguay and exporting to Brazil is going to produce a different good than a firm that decided to locate in Brazil to sell in Brazil. Using the fact that activities are independent, I can calculate profits for each activity separately which simplifies the solution of the problem. Without assumption 1 a firm would have to choose from which location to serve each market. With assumption 1 a firm can serve one market from all the locations. Then, assumption 1 reduces the degree of competition between countries to attract MP. The decrease in competition also reduces the importance of the efficiency of multinationals operating in my country. Without assumption 1, a firm will choose to locate in the country that is more efficient, and the rest of countries will not be able to attract this firm (as long as trade costs are low enough). Now, all countries may attract MP as long as the activity is profitable for a firm. Then the gains I obtained from reducing trade barriers and improving efficiency will be higher without assumption 1 for the most efficient country (the one that will be able able to attract larger amounts of MP).

The importance of assumption 1 is closely link to the role played by BMP. Without assumption 1, BMP would be crucial for the most efficient market to be able to attract MP, specially if the most efficient country is small, since what the firm wants is to serve all countries from the cheapest location (and as opening plants in other countries involve paying a fixed cost, firms may want to minimize the number of locations). The small country, even though is efficient is not going to attract MP since the domestic market is small. Then, BMP becomes a very important factor without assumption 1. Then, my results are a lower bound for the importance of BMP. The importance of BMP for a small country in the open region, like the Netherlands, might be underestimated if the country is used as an export platform to serve the rest of European countries. Also the differences between how much BMP contributes to the gains from openness between Uruguay and the Netherlands (the small countries in each region) will be enhance without assumption 1.

To sum up, assumption 1 simplifies the solution of the problem by making each activity independent. Assumption 1 decreases the competition between countries to attract MP which reduces the importance of the efficiency of multinationals operating in the domestic country. Finally, even though with assumption 1 BMP is an important factor, without assumption 1 BMP will be crucial for attracting MP, specially for small countries.

1.5.3 Firm size distribution

There is a large literature studying the effects of different kind of frictions on the size distribution of firms. Previous studies have focused on the effects of size dependent policies (Guner, Ventura, and Xu (2008), Restuccia and Rogerson (2008), García-Santana and Pijoan-Mas (2012)), capital market imperfections (Erosa (2001), Amaral and Quintin (2010), Buera, Kaboski, and Shin (2011), Greenwood, Sanchez, and Wang (2010)) and trade (Melitz (2003), Piguillem and Rubini (2012)) on firm size distribution. I contribute to these literature by assessing the effect of trade and MP on the distribution of firms' sizes and show that these effects vary across small and large countries within a region, and also among countries of similar size across regions with different degree of openness.

Let first study the total effect of trade and MP in the distribution of firm size. In autarky all countries will have the same distribution of firms,[27] while in the baseline economy this distribution differs significantly across countries. In the baseline economy, the small country in each region has a higher proportion of large firms than the large country. In South America, Uruguay has 1.1% of firms with more than 250 employees while Brazil has 0.8%, and in Europe this proportion is 4.2% for the Netherlands and 1.7% for Italy. It can also be observed that trade and MP has a larger impact on the size distribution of firms for countries in Europe (the open region) than for countries in South America (the closed region). The proportion of firms with more than 250 employees is almost four times larger in the Netherlands than in Uruguay (4.4% vs 1.1%), and in Italy it doubles that from Brazil (1.7% vs 0.8%). As Europe is more open, they benefit more from trade and MP and these shape the distribution of firms increasing the proportion of large firms.

To disentangle the role played by trade and MP in shaping the size distribution of firms, I compute the distribution of firm sizes in a world without MP. Comparing the result of the column No MP to autarky we obtain the contribution of trade to the size distribution of firms, and comparing the result of the column named baseline to the one named No MP we obtain the contribution of MP.

For large countries, MP seems to be the most important factor. While the proportion of large firms is almost unchanged when allowing for trade compared to autarky, it increases significantly when we allow for MP. In Italy from the 1 percentage point increase explained by openness, 0.2 p.p. is explained by trade while 0.8 p.p. is explained by MP. For small countries this is not true. Both trade and MP have similar effects. In Uruguay allowing for trade increases the proportion of large firms 0.2 p.p. and allowing for MP increases the proportion 0.2 p.p. In the Netherlands, trade increases the proportion of large firms by 1.3 p.p. and MP by 2.2 p.p.

[27]This comes from the fact that I am using a Pareto distribution with the same shape parameter for the productivity of firms

To sum up, trade and MP have important effects on the size distribution of firms, but this effect varies across countries and regions. Openness has a larger effect in the size distribution of firms on countries in the open region and in small countries compared to large countries.

1.6 Conclusions

In this paper I construct a heterogeneous firms model of trade with asymmetric countries, MP, and BMP to study the effects of trade barriers and country size in the location decision of multinational firms. I find that BMP is crucial for a small country to attract MP and to take full advantage from trade liberalization and efficiency improvements. BMP explains up to 20% of the gains from openness in the Netherlands while only 10% in Uruguay.

If trade costs are reduced in South America to the average level in Europe, Uruguay's real manufacturing GDP increases 30%. If I do not allow for BMP this increase is reduced by 6 percentage points. If in addition we improve the efficiency of multinationals operating in Uruguay by 20%, real manufacturing GDP increases 41.8%. However, almost all the additional increase in manufacturing real GDP is explained by BMP, since without BMP the increase is 26%, only 2.3 p.p. larger than without any improvement in the efficiency of multinationals.

Finally, MP and BMP shift the distribution of firms toward large firms reinforcing the effect of trade. While in autarky the Netherlands and Uruguay have the same distribution of firms, in the calibrated version of the model, the Netherlands have a proportion of firms with more than 100 employees which doubles that of Uruguay, and with more than 250 employees which is four times larger.

Table 1.1: Calibrated Parameters

Panel A

	L_i	κ^e	κ^d	κ^x	κ^{MP}	z
Argentina	9.47	0.09	1.67	0.34	11.77	0.13
Brazil	48.94	1.95	1.67	1.15	2.07	0.35
Chile	3.69	0.13	1.67	2.05	19.07	0.04
Uruguay	1.00	0.07	1.67	0.82	9.17	0.01
Rest of the World	1582.5	3.00	1.67	1.00	2.67	12.66

Panel B

	L_i	κ^e	κ^d	κ^x	κ^{MP}	z
France	16.8	3.3e-6	1.67	0.89	5.42	0.78
United Kingdom	18.7	3.0e-6	1.67	1.50	10.07	0.90
Italy	14.9	1.0e-6	1.67	1.25	9.87	0.73
Netherlands	4.9	1.0e-6	1.67	3.32	10.97	0.25
Rest of the World	1567.3	1.00	1.67	1.00	2.67	15.67

Table 1.2: Calibration Results-Iceberg Export Costs

Panel A

	Exporting country				
Country	Argentina	Brazil	Chile	Uruguay	RW
Argentina	1	2.27	2.73	2.39	2.61
Brazil	1.48	1	2.36	1.76	2.03
Chile	1.66	2.07	1	2.27	1.93
Uruguay	1.75	2.19	2.57	1	2.68
Rest of the World	1.74	1.97	2.06	2.22	1

Panel B

	Exporting country				
Country	France	UK	Italy	Netherlands	RW
France	1	1.82	1.62	1.74	1.61
UK	1.59	1	1.70	1.50	1.36
Italy	1.80	1.94	1	1.74	1.81
Netherlands	1.52	1.49	1.55	1	1.32
Rest of the World	1.81	1.80	1.78	1.77	1

Table 1.3: Calibration Results-Efficiency of Multinational Firms

Panel A

	Country of origin				
Country	Argentina	Brazil	Chile	Uruguay	RW
Argentina	1	1.47	1.46	1.41	1.48
Brazil	3.75	1	3.08	2.45	2.49
Chile	2.49	2.35	1	2.15	1.81
Uruguay	100	100	100	1	2.02

Panel B

	Country of origin				
Country	France	UK	Italy	Netherlands	RW
France	1	1.62	2.15	1.83	0.33
UK	1.65	1	2.20	1.68	0.28
Italy	1.40	1.49	1	1.55	0.29
Netherlands	1.65	1.47	100	1	0.29

Table 1.4: Performance of the Model-Trade Composition

Trade (as % of Absorption)-Data vs Model

Panel A

	Arg		Bra		Chi		Uru		RW	
	Data	Model	D	M	D	M	D	M	D	M
Arg	-	-	2.9	3.3	3.5	3.8	9.5	9.8	0.2	0.2
Bra	9.6	9.1	-	-	4.2	4.4	17.2	16.8	0.5	0.5
Chi	1.5	1.1	0.5	0.4	-	-	1.8	1.5	0.2	0.1
Uru	0.8	0.9	0.5	0.5	0.4	0.5	-	-	0.0	0.0
RW	24.0	24.2	18.9	18.9	51.3	51.5	29.9	30.1	-	-
Total	35.8	35.1	22.8	23.2	59.4	60.1	58.3	58.2	0.9	0.9

Panel B

	Fra		UK		Ita		Neth		RW	
	Data	Model	D	M	D	M	D	M	D	M
Fra	-	-	8.5	6.3	8.6	7.5	13.8	11.2	2.3	1.7
UK	6.7	7.2	-	-	4.4	5.1	18.6	18.9	2.6	2.8
Ita	7.6	7.2	5.0	4.3	-	-	9.7	10.8	1.7	1.3
Neth	3.4	3.6	5.8	5.3	2.7	3.6	-	-	1.2	1.2
RW	34.2	34.7	49.5	49.7	28.4	28.0	76.2	77.2	-	-
Total	51.9	52.7	68.9	65.6	44.1	44.2	118.4	118.1	7.7	7.1

Table 1.5: Performance of the Model-Foreign Production Composition

Foreign Sales (as % of Total Sales)-Data vs Model

Panel A

	Argentina		Brazil		Chile		Uruguay	
	Data	Model	Data	Model	Data	Model	Data	Model
Argentina	-	-	0.1%	0.1%	0.7%	0.6%	0.0%	0%
Brazil	1.4%	1.5%	-	-	0.3%	0.3%	0.0%	0%
Chile	1.4%	1.4%	0.1%	0.1%	-	-	0.0%	0%
Uruguay	1.1%	1.0%	0.2%	0.2 %	0.3%	0.3%	-	-
RW	31.9%	31.7%	7.8%	8.0%	32.5%	32.3%	29.7%	30.7%

Panel B

	Fra		UK		Ita		Neth	
	Data	Model	Data	Model	Data	Model	Data	Model
France	-	-	2.5%	2.1%	2.5 %	2.6%	2.9%	2.6%
UK	3.0%	3.4%	-	-	1.6%	1.9%	4.9%	5.3%
Italy	1.6%	2.1%	1.2%	1.3%	-	-	0.0%	0.0%
Netherlands	1.4%	1.4%	2.1 %	1.6 %	1.3%	1.2	-	-
RW	20.1%	20.3%	38.9%	37.8%	13.2%	12.7%	35.0%	35.1%

Table 1.6: Calibration Results: Aggregate Targets

Panel A

	Data vs Model							
	% Exporting Firms		% Foreign Firms		GDP per Capita		Trade Balance	
	Data	Model	D	M	D	M	D	M
Arg	52.3	52.4	7.9	7.9	1.56	1.56	-6.0	-6.1
Bra	14.1	14.1	7.2	6.9	0.87	0.86	-1.5	-1.8
Chi	24.6	24.3	5.8	5.8	1.08	1.08	-8.9	-8.8
Uru	33.3	32.8	7.7	8.0	1.22	1.24	-10.1	-9.9

Panel B

	Data vs Model							
	% Exporting Firms		% Foreign Firms		GDP per Capita		Trade Balance	
	Data	Model	D	M	D	M	D	M
Fra	44.7	45.0	11.5	11.7	4.7	4.6	0.7	0.7
UK	37.0	36.7	12.6	12.8	4.6	4.6	-2.2	-2.4
Ita	28.4	28.9	3.6	3.9	5.5	5.5	8.9	9.5
Neth	42.2	42.5	12.9	12.5	5.0	5.0	-1.8	-1.6

Table 1.7: Experiment Results-Closing the Economies

Panel A

	Changes in %			
	Autarky with BMP		Autarky without BMP	
	Real GDP	Real GNP	Real GDP	Real GNP
South-America	-5.3	-3.4	-5.0	-3.5
Argentina	-9.5	-4.9	-9.0	-5.2
Brazil	-3.6	-2.5	-3.5	-2.5
Chile	-11.9	-8.7	-10.9	-8.9
Uruguay	-12.1	-10.8	-10.9	-10.4

Panel B

	Changes in %			
	Autarky with BMP		Autarky without BMP	
	Real GDP	Real GNP	Real GDP	Real GNP
Europe	-10.5	-7.3	-9.3	-7.3
France	-9.1	-6.4	-8.3	-6.3
UK	-13.4	-8.8	-11.9	-9.0
Italy	-5.6	-3.5	-5.1	-3.6
Netherlands	-20.3	-17.1	-17.0	-16.5

Table 1.8: Experiment Results-The Effects of MP and BMP

Panel A

	Relative losses in real GDP		
	$\frac{\text{Baseline}}{\text{World without BMP}}$	$\frac{\text{World without BMP}}{\text{World without MP}}$	$\frac{\text{Baseline}}{\text{World without MP}}$
Argentina	1.055	2.033	2.145
Brazil	1.035	1.510	1.563
Chile	1.099	1.478	1.623
Uruguay	1.112	1.510	1.680

Panel B

	Relative losses in real GDP		
	$\frac{\text{Baseline}}{\text{World without BMP}}$	$\frac{\text{World without BMP}}{\text{World without MP}}$	$\frac{\text{Baseline}}{\text{World without MP}}$
France	1.087	1.679	1.826
UK	1.128	1.671	1.885
Italy	1.097	1.841	2.019
Netherlands	1.199	1.203	1.443

Table 1.9: Experiment Results-Reducing Costs

Panel A

	Changes (in %)			
	Same MP costs		Same MP Costs-No BMP	
	Real GDP	Real GNP	Real GDP	Real GNP
South-America	6.2	6.1	5.3	6.0
Argentina	11.2	11.1	9.0	11.7
Brazil	4.0	4.0	3.7	4.0
Chile	13.1	12.4	9.4	12.4
Uruguay	29.9	29.1	23.7	27.1

Panel B

	Changes (in %)			
	Improve 20% efficiency		Improve 20% efficiency- No BMP	
	Real GDP	Real GNP	Real GDP	Real GNP
South-America	9.7	6.6	7.6	6.9
Argentina	17.7	11.7	13.0	13.0
Brazil	6.3	4.3	5.6	4.5
Chile	21.4	14.3	12.4	14.3
Uruguay	38.3	36.6	24.4	30.7

Panel C

	Changes (in %)			
	Improve 20% efficiency only in Uruguay		Improve 20% efficiency only in Uruguay- No BMP	
	Real GDP	Real GNP	Real GDP	Real GNP
South-America	6.3	6.1	5.3	6.2
Argentina	11.1	11.1	9.0	11.7
Brazil	4.0	4.0	3.7	4.0
Chile	13.1	12.4	9.4	12.4
Uruguay	41.8	29.3	26.0	28.1

For all the experiments I use the average trade costs in Europe ($\tau = 1.64$)

Table 1.10: Experiment Results-Firms Size Distribution

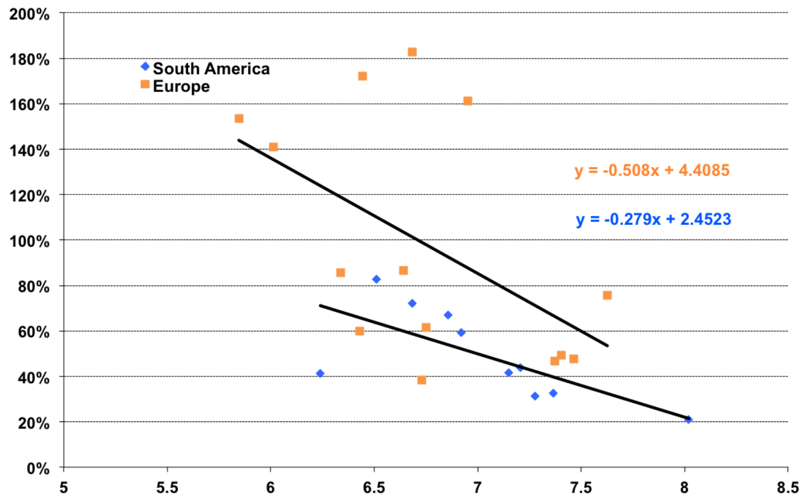
Panel A

	Proportion of firms with more than x employees							
	Benchmark		No BMP		No MP		Autarky	
	> 100	> 250	> 100	> 250	> 100	> 250	> 100	> 250
Uruguay	3.4	1.1	3.2	1.0	2.9	0.9	2.2	0.7
Netherlands	7.9	4.2	7.5	3.7	5.6	2.0	2.2	0.7

Panel B

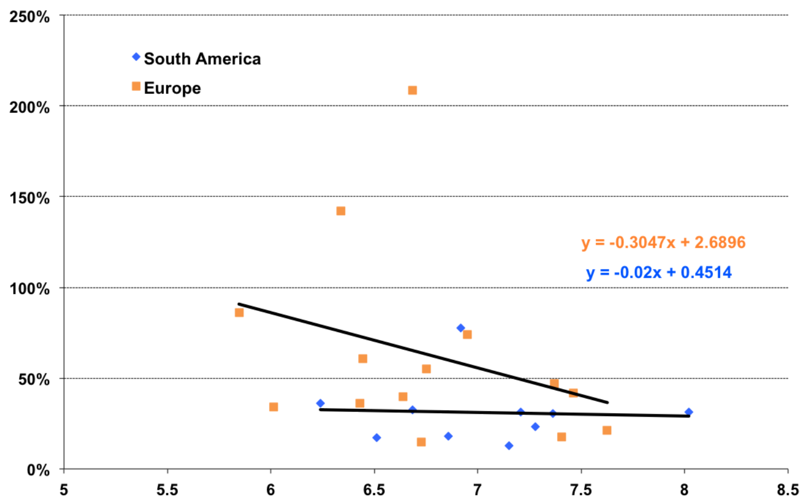
	Proportion of firms with more than x employees							
	Benchmark		No BMP		No MP		Autarky	
	> 100	> 250	> 100	> 250	> 100	> 250	> 100	> 250
Brazil	2.4	0.8	2.3	0.8	2.4	0.8	2.2	0.7
Italy	3.5	1.7	3.5	1.6	2.8	0.9	2.2	0.7

Figure 1.1: Correlation $\frac{Trade}{GDP}$ and Size



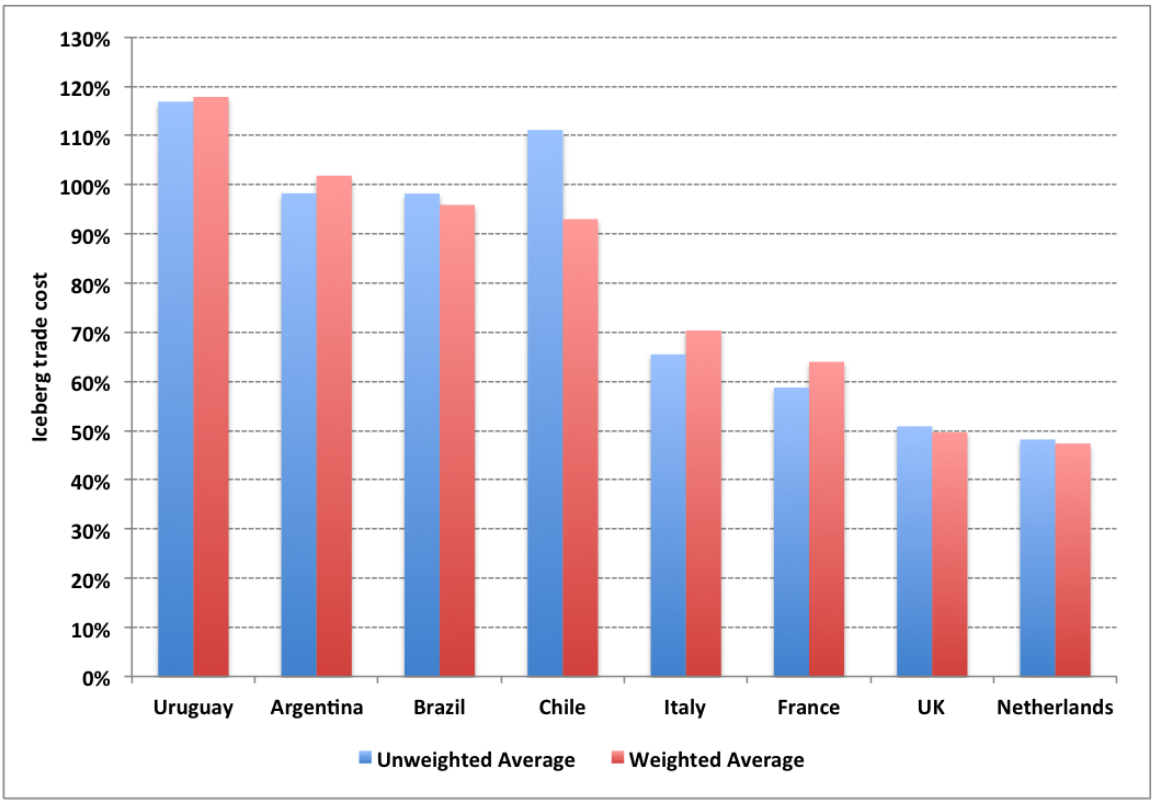
Source: Author's elaboration based on UNCTAD data for 2012

Figure 1.2: Correlation $\frac{FDIStock}{GDP}$ and Size



Source: Author's elaboration based on UNCTAD data for 2012

Figure 1.3: Average iceberg trade costs



Chapter 2

Financial Frictions, Occupational Choice and Economic Inequality

2.1 Introduction

A recent literature has emphasized that the misallocation of resources caused by financial frictions depress total factor productivity and, hence, output per worker (Erosa (2001), Jeong and Townsend (2007), Amaral and Quintin (2010), Buera and Shin (2011), Buera, Kaboski, and Shin (2011), Greenwood, Sanchez, and Wang (2010)). The standard approach in the financial frictions literature and, more generally in the misallocation literature (see Guner, Ventura, and Xu (2008), and Restuccia and Rogerson (2008)) is to calibrate the model to micro data from the United States and use the calibrated model economy to simulate policy distortions in developing countries.[1] While this approach has the advantage that the US data is readily available, it relies on the assumption that the distribution of entrepreneurial skills or plant productivities are invariant across countries or, at the very least, do not matter for the misallocation of resources induced by policy distortions or limited enforcement in the financial markets. However, there is ample evidence suggesting that the distribution of skills do vary across rich and poor countries.[2] Moreover, economic theory suggests that inequality matters for the impact of micro distortions and financial frictions (see Banerjee and Newman (1993), Galor and Zeira (1993)).

We develop a quantitative theory of entrepreneurship, income inequality, and financial frictions disciplined with household level data from Brazil. The theory is used to quantitatively evaluate the impact of financial frictions on occupational decisions, resource allocation, aggregate output, and economic inequality. Conversely, we study how economic inequality shapes the impact of

[1] Notable exceptions are given by Midrigan and Xu (2014) and García-Santana and Pijoan-Mas (2012)

[2] In fact, even among developed economies, recent work on international trade theory argues that the heterogeneity in the (second moments of the) skill distribution plays an important role for understanding trade patterns among similarly endowed economies (see Ohnsorge and Trefler (2007) and references in that paper).

financial frictions in the economy. Our paper contributes to a seminal (mostly theoretical) literature that has emphasized the importance of the interaction between the distribution of wealth and financial frictions for the allocation of resources. Moreover, we use our theory of inequality to quantitatively assess the distribution of welfare gains and losses from eliminating financial frictions in the economy.

The key innovation of our theory is to extend the Lucas (1978) model in order to incorporate heterogeneity in two skills: −working and managerial skills. By modeling heterogeneity in two skills the theory can distinguish between comparative advantage in entrepreneurship (a high ratio of managerial to working skills) and absolute advantage (a high value of both skills). This distinction is necessary for the theory to be consistent with evidence on the income distribution across occupations in Brazil. In particular, the ratio of median earnings between entrepreneurs and workers in Brazil is equal to one, which implies that the median entrepreneur in the Brazilian data does not make higher earnings than the median worker. This implication is grossly at odds with the predominant one-skill model developed in the occupational choice literature.

By assuming that entrepreneurs can use their working and managerial skills in the operation of their businesses, our theory has the novel implication that some entrepreneurs will not hire any outside labor and be own account workers (or self-employed entrepreneurs). Building a theory that distinguishes between entrepreneurs that are employers and those that are self-employed is important because there is abundant evidence that the high rates of entrepreneurship in poor countries is mostly due to the prevalence of self-employed workers (see Figure 2.2). In fact, the high rates of self-employment is an important feature of the Brazilian data and, our findings, imply that self-employment is important for understanding the impact of financial frictions in Brazil.

We assume a small open economy that takes as given the international interest rate. Following Buera, Kaboski, and Shin (2011), capital market imperfections are introduced by modeling an endogenous borrowing constraint that limits the amount of capital that entrepreneurs can use. We prove that in the absence of financial frictions occupational choices are driven entirely by the ratio of managerial to working skills. Employers have a comparative advantage at managing (high $\frac{z_m}{z_w}$), workers have a comparative advantage at working (low $\frac{z_m}{z_w}$), and self-employed have an intermediate skill ratio. Heterogeneity in absolute advantage implies that both at the top and bottom of the income distribution there are entrepreneurs and workers. We characterize how capital market imperfections distort rates of returns on skills by making the return to managerial and working skills depend on asset holdings. We show that financial frictions have a non-trivial impact on inequality: On the one hand, they lead to higher and persistent inequality by generating variation in returns to skills and by making these returns depend on asset holdings. On the other hand, since borrowing constraints tend to be tighter for highly skill than for low skill individuals, financial frictions reduce inequality by diminshing the rents obtained by highly skilled individuals relative to a situation with perfect capital markets.

The model economy is calibrated to Brazilian household data and macro aggregates. Brazil provides a nice benchmark because it is a country that exhibits both high levels of economic inequality and of financial frictions. We simulate the effects of removing financial frictions in the calibrated model economy. We find a large drop in the rates of entrepreneurship (from 33% to 18%), which is mostly due to a decrease in self-employment. While the self-employment rate decreases from 24% to 11%, the fraction of employers only drops by about 2 percentage points. The gain in aggregate output amounts to 48%. There are also sizable changes in the sectorial composition of output. Production by employers increase by 64% whereas production of the self-employed decreases by 53%. TFP in the economy increased by 24%, with this gain being larger than the TFP gain among self-employed (9%) and that among employers (16%). The share of aggregate capital used by the self-employed drops from 19% to 4%. Because employers tend to have a higher managerial ability than self-employed entrepreneurs, the reallocation of capital between these two groups enhances the aggregate productivity gains of removing financial frictions.

We also simulate the impact of removing financial frictions when self-employment is shut down in the baseline economy. We find that the output gain is 53%, which is higher than the 48% increase obtained in the baseline economy. Hence, self-employment decreases the negative impact of financial frictions on aggregate output. This finding can be explained as follows: Financial frictions make it hard for young and talented entrepreneurs (individuals with high managerial skills) to raise external funds. This effect is compounded by the fact that financial frictions depress the equilibrium real wage, which makes it difficult to accumulate savings by working for a paid wage when young. Self-employment allows talented entrepreneurs to circumvent the low wage and build up savings, diminishing the negative impact of financial frictions on aggregate output. Hence, our findings implies that self-employment diminishes the impact of financial frictions by being a pathway towards becoming an employer. The occupational transitions in the Brazilian household data support this prediction of the theory: Self-employed individuals are three times more likely to become employers than paid workers and about 40% of transitions into employer between two consecutive years are coming directly from self-employment. Our baseline economy matches these facts remarkably closely.

Financial frictions have important effects on the sources of income inequality and on its persistence over time. We divide household income between capital income and labor income, with labor income defined as the sum of the returns to working and managerial skill inputs. Surprisingly, we find that capital market imperfections have opposing effects on the concentration of labor income and capital income. Labor income is more evenly distributed in the economy with imperfect capital markets than in the economy with no financial frictions (with a Gini index of

.52 versus a Gini index .56 in the latter economy). This is because financial frictions depress the rents earn by highly able entrepreneurs relative to an economy with perfect capital markets . On the other hand, the Gini index of capital income is about 10 percentage points higher in the economy with imperfect credit markets. Financial frictions imply that the returns to managerial ability are positively correlated with capital income. The correlation between capital and labor income is equal to 0.80 in the baseline economy, which is much larger than the 0.50 value in the economy with perfect capital markets. This effect explains why income inequality is also more persistent in the economy with financial frictions.

The skill distribution matters importantly for the impact of financial frictions. This is shown by recalibrating two new economies in which the correlation between working and managerial skills is set exogenously to a high (0.8) and a low (-0.8) value (the calibration of the baseline economy implies a skill correlation of 0.1). We find that the output and TFP gains of improving credit market institutions are large in all economies but vary substantially across the three calibrated model economies. The output gains range from 36% to 55% and the TFP gains range from 22% to 31% as the correlation between skills decreases. The skill correlation determines the extent to which talented entrepreneurs are able to self-finance their businesses. When the correlation between these two skills is high, individuals that are talented as entrepreneurs are also talented as workers. Then, if skills are also persistent over time, young and talented individuals can work when young, build savings, and use their savings to finance their businesses when old. Thus, when managerial and working skills are highly correlated and persistent over time, the effects of financial frictions on resource allocations are less important than otherwise.

Given that financial frictions are so detrimental for the efficient allocation of productive resources and aggregate output, one question that rises is why countries set up institutions causing financial frictions and why these institutions are so persistent. Acemoglu and Robinson (2012) provided many historical accounts of how political power determines economic institutions and, in turn, how political power is shaped by the political institutions and the distribution of resources in society. Given that our theory of inequality was calibrated to Brazilian household data we can gain some insights into the political economy of capital market imperfections by studying the welfare gains and losses of reforming capital markets in our calibrated model economy. We assume that the economy is in steady state and that suddenly there is a once and for all reform that makes the enforcement of credit contracts perfect. We find that the financial reform has non-trivial effects on the distribution of income and that occupational choices are crucial for understanding how the reform impacts on individuals. Keeping fixed the occupational choices of the initial steady state, workers gain with the reform since the wage rate increase. Self-employed also gain since they can borrow more. Unconstrained employers loose since wage payments go up but their managerial rents do not increase. Constrained employers may gain or loose depending on whether the increase in managerial rents outweighs the increase in wages. Since the increase in managerial rents is likely to be more important for talented entrepreneurs, these entrepreneurs are more likely to see their income increase. The untalented entrepreneurs who were operating businesses prior to the reform are likely to see their income go down and to switch occupations.

We find that while the vast majority of households gain from the reform, about 8.7% of the population see their welfare decrease with the reform. Households that lose from the reform tend to be older, richer, and exhibit higher managerial skills and lower working skills than households that support the reform. These findings are just reflecting that occupational choices are crucial for understanding the political economy of the reform: Among the households that are worse off with the reform, about 93% of them would have been entrepreneurs on the period of the reform had the reform not taken place, and 66% would have been employers. Employers are a positive selection from the population distribution of managerial skills. Then, the fact that about two thirds of those who oppose the reform are employers explains why the managerial ability of those supporting the reform is higher than that of those opposing the reform. Nonetheless, not all employers support the reform: About 36% of employers in the initial equilibrium benefit from the elimination of enforcement problems. We find that the employers benefiting from the reform tend to be of higher managerial ability than those who oppose it. The reason is that high ability employers are more likely to be borrowing constrained than low ability entrepreneurs. As a result, they are more likely to operate at an inefficient scale and to gain more from the elimination of enforcement problems. On the other hand, the financial reform hurts many of the lower skill employers and force them to change their occupation status: About 46% of the entrepreneurs that oppose to the reform and would have been employers had the reform not taken place, do not hire any labor after the reform (most of them become self-employed after the reform). The wage hike after the reform makes it unprofitable for these entrepreneurs to hire outside labor.

Summing up, while most households benefits from a reform that eliminates enforcement problems, the majority of employers (about two thirds) lose from the reform. By depressing the demand for labor, limited enforcement depresses the equilibrium wage rate, increasing the profits of employers. Our theory thus suggests that employers may have a vested interested in maintaining a status quo with low enforcement.

The paper continues as follows. Section 2 presents some facts on entrepreneurship and economic inequality in Brazil. Section 3 presents the model economy. Section 4 presents some analytical results characterizing how financial frictions affects occupational choice decisions, rates of returns to skills, and inequality. Section 5 calibrates the model economy, evaluates the performance of the model economy, and assesses the effects of removing financial frictions on occupational choices, aggregate output, and income inequality. This section also discusses how changes in the correlation of skills affect the impact of financial frictions on aggregate output. The paper ends with a discussion of the political economy of removing financial frictions in Brazil.

2.2 Evidence

We now document some facts on occupations and economic inequality in Brazil that guide the theory developed in this paper. The facts are based on data from the *Pesquisa Mensal de Emprego* (PME) and from the *Pesquisa de Ornamentos Familiares* (POF). The former is a monthly household employment survey, with a similar structure to the US Current Population Survey (CPS). The latter is a survey of household consumption and income. Appendix A describes how the data set used in this paper was constructed.

Income inequality Figure 2.1 presents data on the variance of log-income over the life cycle from the PME (similar findings arise from the POF). First, note that the variance of log-income at age 20 is 0.55, which is much higher than the value of 0.30 documented by Storesletten, Telmer, and Yaron (2005) for the United States. Thus, households in Brazil are quite heterogeneous at young ages. As in the United States, inequality in income grows during the life cycle suggesting the presence of persistent shocks to household earnings. By age 55, household log-income reaches a value of 1.01.

Occupational structure We define the occupation of a household as that of the household head. We consider two broad occupations – workers and entrepreneurs. Moreover, we further subdivide the entrepreneurial occupation in two classes – employers and own account workers (self-employed). Figure 2.2 uses data from the ILO to analyze occupational structure in different countries. The blue bar on Figure 2.2 shows the proportion of workers, the orange bar the proportion of self-employed, and the green bar the proportion of employers.[3] The evidence shows that developed countries have lower amount of entrepreneurs than developing countries, but this data pattern is driven by the lower proportion of self-employed in developed countries. The proportion of employers in the population of households is quite similar among countries. While in Brazil workers represent about 73% of households, in Germany they are about 89%. The high proportion of entrepreneurial households in Brazil is explained by self-employed households which represent about 22% of the labour force in Brazil, much lower than the 6% of self-employed households in Germany. The fraction of employers is roughly equal across these two countries (about 5%). Employers and self-employed are quite different in their average income: On average employers earn about 3 times as much as self-employed households. Moreover, self-employed earn less than the average worker.

Distribution of earnings by occupation We show two graphs on the distribution of earnings by occupation: The first one shows the distribution of earnings for workers and

[3]For Canada and the United States, the ILO does not distinguish between self-employed and employers so that the orange bar is the sum of the two.

entrepreneurs, the second one shows the earnings for entrepreneurs partitioned between self-employed and employers. Figure 2.3 shows that the distribution of earnings of entrepreneurs is flatter than the one of workers, having a bigger mass of people with low earnings but also a bigger mass of households with high earnings. Thus, earnings are more dispersed among entrepreneurs than workers. If we further divide the Entrepreneurs in Self-Employed and Employers we can see that the first group is the one that has more mass in the lower tail of earnings. Figure 2.4 shows that Self-Employment is the occupation with the lowest expected returns, while Employer is the one with the highest expected returns.

Summarizing, we draw the following lessons from the above facts:

1. Income inequality in Brazil is high relative to the US, which underscores the importance of calibrating the model to Brazilian micro data. Brazilian households are highly heterogeneous early in the life cycle and inequality grows substantially with age. These observations suggests the importance of modeling heterogeneity in fixed effects (permanent skill heterogeneity) as well as persistent shocks to skills.

2. The fact that both wages and entrepreneurial income are highly dispersed, motivates us to build a model with two dimensional skill heterogeneity.

3. It is important to build a theory that distinguishes between employers and self-employed entrepreneurs since most entrepreneurial households in Brazil are self-employed (or own-account workers) households and distribution of income differ substantially across both categories of entrepreneurs. While mean income of employers is much higher than that of self-employed households, there is substantial income heterogeneity within each of these occupational categories.

4. The variation in the rates of entrepreneurship between Brazil and rich countries is entirely explained by the high rates of self-employment in Brazil, a fact that existing theories of occupational choice cannot account for.

2.3 The Model

We consider a small open economy in steady state. The model features a one sector life-cycle growth model in which households are heterogeneous in two skills – working (z_w) and managerial abilities (z_m). Skills evolve stochastically over the life cycle and there are no insurance markets to insure ability risk. Production is organized by entrepreneurs who combine managerial, capital, and labor inputs. As in Lucas (1978), entrepreneurs can only use their own managerial skills since there is no markets for managers. In each period households choose their occupation:

whether to work for a wage or to operate a business and become entrepreneurs. Occupational choices are based on their comparative advantage as entrepreneurs and their access to capital. Following Buera, Kaboski, and Shin (2011), access to capital is limited by their wealth through an endogenous collateral constraint that arises because of enforcement problems. In order to match important aspects of the Brazilian micro data, the Lucas (1978) model is extended to distinguish between two types of entrepreneurial households – employers and self-employed households.

Population The economy is populated by overlapping generations, each generation consisting of a continuum of households. Households are born at age 20, retire at age 60, and die with certainty at age 75. Each households is endowed with one unit of time at every age. Before the retirement age, households decide how much of their time to allocate to working (t_w) or to managerial (t_m) activities. Households differ in working (z_w) and managerial (z_m) abilities. The logarithm of skills evolve stochastically over the life cycle according to (household i at age t)

$$\begin{aligned} ln(z_{wit}) &= \beta_w X_t + \alpha_{wi} + u_{wit}, \\ ln(z_{mit}) &= \beta_m X_t + \alpha_{mi} + u_{mit}, \end{aligned}$$

where z_{wit} (z_{mit})denote the working (managerial) skills of household i at age t, X_t represents a quartic polynomio of age, α_{wi} and α_{mi} represent household fixed effects on working and managerial productivities, and u_{wit} and u_{mit} are life cycle shocks received at age t by household i. We assume that the fixed effects are drawn from a bi-variate normal distribution at the first period of life of the household (age 20):

$$\alpha = (\alpha_{wi}, \alpha_{mi}) \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_w^2 & \rho\sigma_w\sigma_m \\ \rho\sigma_w\sigma_m & \sigma_m^2 \end{bmatrix} \right)$$

where ρ is the correlation between the two fixed effects across individuals. The mean fixed effect of the distribution of working skills is normalized to 0.

The life-cycle shocks follow the stochastic process

$$u_{jit} = \rho_j u_{jit-1} + \epsilon_{jit}, \text{ for } j = w, m,$$

with $\epsilon_t = (\epsilon_{wt}, \epsilon_{mt})$ jointly drawn from a bivariate normal distribution with correlation coefficient $corr(\epsilon_{wt}, \epsilon_{mt}) = \rho$. We further assume that α_{ji} and u_{jit} are mutually orthogonal.

The assumptions made imply that distribution of skills at age-t is log-normally distributed

$$\begin{aligned}
\begin{pmatrix} ln(z_{wt}) \\ ln(z_{mt}) \end{pmatrix} &\sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2_{wt} & \rho_{wmt}\sigma_{wt}\sigma_{mt} \\ \rho_{wmt}\sigma_{wt}\sigma_{mt} & \sigma^2_{mt} \end{bmatrix}\right) \\
\sigma^2_{wt} &= \sigma^2_{\alpha_w} + \sum_{j=0}^{t-1}(\rho^j_w)^2\sigma^2_{\epsilon_w} \\
\sigma^2_{mt} &= \sigma^2_{\alpha_m} + \sum_{j=0}^{t-1}(\rho^j_m)^2\sigma^2_{\epsilon_m} \\
\rho_{wmt}\sigma_{wt}\sigma_{mt} &= cov(\alpha_w, \alpha_m) + \sum_{j=0}^{t-1}\rho^j_w\rho^j_m cov(\epsilon_w, \epsilon_m) \\
cov(\alpha_w, \alpha_m) &= \rho\sigma_{\alpha_w}\sigma_{\alpha_m} \\
cov(\epsilon_w, \epsilon_m) &= \rho\sigma_{\epsilon_w}\sigma_{\epsilon_m}
\end{aligned}$$

Production technology Following Lucas (1978), output is produced with a constant returns to scale production technology in managerial, labor, and capital inputs. Entrepreneurs can only use their managerial input because there is no market for managers. The supply of the managerial input is equal to the product of the households' managerial ability (z_m) and the time devoted to managing a business (t_m). The output produced by a household supplying $m = z_m t_m$ units of managerial input and using k units of capital and n efficiency units of labor is:

$$Y(m, k, n) = m^\gamma k^\nu n^\theta, \text{ where } \gamma + \nu + \theta = 1. \tag{2.1}$$

The time allocation decision of entrepreneurs ($t_m \in [0, 1]$) is modeled to introduced self-employment in the Lucas (1978) framework. When $0 < t_m < 1$ entrepreneurs supply *both* managerial and labor inputs to their own businesses. Specifically, the labor input supplied by entrepreneurs to their business is equal to the product of their working ability (z_w) and the time devoted to non-managerial activities ($1 - t_m$). The total labor input used by an entrepreneur is the sum of the labor supplied by the entrepreneur ($(1 - t_m)z_w$) and the labor hired in the market (n^d) from workers outside the family:

$$n = n^d + (1 - t_m)z_w, \tag{2.2}$$

where z_w is the working ability of the household. We denote as entrepreneurs the households that choose $t_m > 0$. Entrepreneurs, in turn, are partitioned in two subgroups depending on whether they hire outside labor or not. The first subgroup is given by the employers, who are those entrepreneurs hiring labor outside the family ($n^d > 0$) . We assume that entrepreneurs that hire outside labor incur a fixed per period operating cost of c_f.[4] The second subgroup are

[4]The fixed cost is introduced so that employers demand a non-trivial amount of labor (an amount bounded away from zero), thereby making the distinction between self-employed and employer meaningful.

those entrepreneurs that only use their own household labor input ($n = (1 - t_m)z_w$ and $n^d = 0$). Workers are those households who use all their available time as workers ($t_m = 0$, obtaining labor earnings wz_w).

Summarizing, entrepreneurs produce output with a production technology that combines capital, labor, and managerial inputs. The key distinguishing feature between employers and self-employed is that the latter do not hire labor outside the household and that employers pay a fixed cost in each period of business operation. They both solve a time-allocation problem regarding the fraction of their time endowment used to supply managerial versus working skills. Below, we shall characterize how entrepreneurs optimally choose the time (t_m) dedicated to the supply of managerial skills.

Capital markets We assume that the financial intermediation industry is competitive. Intermediaries take deposits from households and pay the international interest rate r. They rent capital to entrepreneurs at a rate $r + \delta$ and loan employers the fixed cost of operation c_f. Enforcement problems limit the amount of borrowing and the capital rented to entrepreneurs. Following Buera, Kaboski, and Shin (2011), entrepreneurs may renege on the contracts after production has taken place and keep a fraction $1 - \phi$ of undepreciated capital and the revenue net of labor payments $(Y(m, k, n) - wn^d + (1 - \delta)k - c_f \mathrm{I}_{n_d>0})$ but lose the financial assets a deposited with the intermediary. Entrepreneurs that default regain access to the financial markets the following period. The parameter $\phi \in [0, 1]$ indexes the strength of the legal institutions in the economy, with $\phi = 1$ indicating perfect financial markets and $\phi = 0$ corresponding to an economy with no credit markets. We study equilibria in which financial contracts are restricted so that there is no default in equilibrium. This occurs when the amount of capital rented is limited by the largest amount $\overline{k}(a, z_m, z_w; \phi)$ consistent with entrepreneurs choosing to abide by their financial contracts. To characterize rental limits, consider the profit maximization problem of entrepreneurs that take as given the capital k used in the business operation:

$$\pi(z_m, z_w, a; k) \equiv \max_{m,n,n_d,t_m} \{m^\gamma k^\nu n^\theta - wn^d - r(k - a) + a - \delta k - c_f I_{n_d>0}\} \tag{2.3}$$

$$\textit{subject to}$$

$$m = t_m z_m, \tag{2.4}$$

$$n = (1 - t_m) * z_w + n^d, \tag{2.5}$$

$$\textit{where } t_m \in [0, 1], n^d \geq 0, k \textit{ given.} \tag{2.6}$$

The following proposition extends results in Buera, Kaboski, and Shin (2011) to characterize the rental limits $\overline{k}(a, z_m, z_w; \phi)$.

Proposition 1 *Capital rental k by an entrepreneur with wealth a and skills (z_m, z_w) is enforceable if and only if*

$$\begin{aligned} \pi(z_m, z_w, a; k) \quad \geq \quad & (1-\phi) \max_{m,n,n_d,t_m} \{m^\gamma k^\nu n^\theta - wn^d + (1-\delta)k - c_f I_{n_d>0}\} \\ & \textit{subject to} \\ & m = t_m z_m, \\ & n = (1-t_m) * z_w + n^d, \\ & \textit{where } t_m \in [0,1], n^d \geq 0. \end{aligned}$$

The upper bound on capital rental that is consistent with entrepreneurs choosing to abide by their contracts can be represented by a function $\overline{k}(a, z_m, z_w; \phi)$, which is increasing in a, z_m, z_w and ϕ.

Proof. See appendix.

The income of an entrepreneur in state (z_m, z_w, a) making optimal production decisions given prices and borrowing limits is given by

$$y^e(z_m, z_w, a) \quad \equiv \quad \max_k \{\pi(z_m, z_w, a; k)\} \tag{2.7}$$

$$\textit{subject to}$$

$$k \leq \overline{k}(a, z_m, z_w; \phi) \tag{2.8}$$

The income of a household that choose to work for a wage is $y^w(z_m, z_w, a) = wz_w + ra$. Household income is the maximum between the entrepreneurial and workers income:

$$y(z_m, z_w, a) = \max\{y^e(z_m, z_w, a), y^w(z_m, z_w, a)\}. \tag{2.9}$$

Households maximize expected discounted lifetime utility

$$\begin{aligned} \max_{c_j, a_{j+1}} \quad & E\{\sum_{j=1}^{J} \beta^j U(c_j)\} \\ \textit{subject to} & \\ c_j + a_{j+1} \quad = \quad & y(z_{mj}, z_{wj}, a_j), \\ c_j, a_{j+1} \quad \geq \quad & 0, \end{aligned}$$

2.4 Time Allocation and Occupational Maps

We now study in partial equilibrium (e.g. for a fixed wage rate) how our theory can give rise to three active occupational choices: workers, self-employed, and employers. We show that when capital markets are perfect occupational choices are entirely determined by the ability ratio $\frac{z_w}{z_m}$. Individuals with a high $\frac{z_w}{z_m}$ ratio have a comparative advantage at working and choose to become workers, individuals with a low $\frac{z_w}{z_m}$ ratio have a comparative advantage at entrepreneurship and choose to become employers, and those with intermediate skill ratios prefer to be self-employed. We also characterize how tight borrowing constraints (capital market imperfections) distort occupational choices.

We start by analyzing the determinants of self-employment income. Self-employed individuals choose how much time to allocate to managerial versus working activities and how much capital to use in production. Using the linear homogeneity of the production function, the income of a self-employed individual with a units of assets who uses k units of capital can be written as

$$y_{se} = MPt_m\, t_m + MPt_w\, t_w + MPK\, k + ra - k(r+\delta),$$

where MPt_m and MPt_w denote the marginal products of managerial time and working time, respectively, and MPK represents the marginal product of capital. We are now ready to prove the following proposition:

Proposition 2 (Self-employment) *The optimal time devoted to management by self-employed entrepreneurs (entrepreneurs not hiring outside labor) is $t_m^* = \frac{\gamma}{\gamma+\theta}$. The marginal product of their time is equated across its two uses (managerial and working time) and satisfies:*

$$MPT_{se} = r_{mw}\left(z_m^{\gamma} z_w^{\theta}\right)^{\frac{1}{\gamma+\theta}},$$

where $r_{mw} = \gamma\, \nu^{\frac{\nu}{1-\nu}} \left(\frac{\theta}{\gamma}\right)^{\frac{\theta}{1-\nu}} \left(\frac{1}{r+\delta+\mu}\right)^{\frac{\nu}{1-\nu}}$ is the rate of return to the composite skill input $\left(z_m^{\gamma} z_w^{\theta}\right)^{\frac{1}{\gamma+\theta}}$ and μ is the Lagrange multiplier associated to the borrowing constraint. The income of a self-employed individual with assets a is given by

$$y_{se} = r_{mw}\left(z_m^{\gamma} z_w^{\theta}\right)^{\frac{1}{\gamma+\theta}} + \mu k + ra,$$

where $k = \overline{k}(z_m, z_w, a)$.

Proof. See appendix.

Proposition 2 establishes that self-employed individuals equate the marginal product of the time allocated to managing and to working tasks. The marginal product of the self-employment time can be expressed as the product of the skill composite $\left(z_m^\gamma z_w^\theta\right)^{\frac{1}{\gamma+\theta}}$ and the rate of return r_{mw}. The skill composite $\left(z_m^\gamma z_w^\theta\right)^{\frac{1}{\gamma+\theta}}$ is a geometric average of the managerial and working abilities of the self-employed individual. The return to the skill composite (r_{mw}) depends on parameters of the production technology, the real interest rate (r), and the Lagrange multiplier (μ) associated to the borrowing constraint. Note that the return to the skill composite decreases with (μ). Hence, borrowing constraints generate heterogeneity in rate of returns to skills among self-employed individuals.

Since workers' income is given by

$$y_w = wz_w + ra,$$

it is immediate that $y_{se} - y_w$ is independent of asset holdings. Hence, as shown in Proposition 3, when $\mu = 0$ the decision of whether to work for a wage or to be self-employed only depends on the ability ratio $\frac{z_w}{z_m}$. On the contrary, when the borrowing constraint binds $k = \overline{k}(z_m, z_w, a)$ occupational choice decisions depend on asset holdings because they affect the rate of return to skills (e.g. the composite input $\left(z_m^\gamma z_w^\theta\right)^{\frac{1}{\gamma+\theta}}$) and the rate of return to assets. These results are summarized in Proposition 3.

Proposition 3 (Self-employed vs Worker) *Let* $R_1 \equiv \left(\frac{r_{mw}}{w}\right)^{\frac{\theta+\gamma}{\gamma}}$ *, where* r_{mw} *is defined in Proposition 2. Then,*

1. *If capital markets are perfect ($\phi = 1$), working for a wage is preferred to self-employment if and only if* $\frac{z_w}{z_m} > R_1$.
2. *If capital markets are imperfect ($\phi < 1$), working for a wage is preferred to self-employment if and only if*

$$\frac{z_w}{z_m} > \left[\frac{r_{mw} + \mu k / \left(z_m^\gamma z_w^\theta\right)^{\frac{1}{\gamma+\theta}}}{w}\right]^{\frac{\theta+\gamma}{\gamma}} \equiv \overline{R}_1,$$

where $\mu > 0$ is the Lagrange multiplier associated to the borrowing constraint and $k = \overline{k}(z_m, z_w, a)$.

When capital markets are perfect ($\phi = 1$), the occupational choice decision between working for a wage or being self-employed can be represented by a ray R_1 that goes through the origin in (z_m, z_w) space. Individuals with ability above this ray prefer to be a worker. In this case, occupational choice decisions are independent of asset holdings and maximize the marginal product of time. However, when capital markets are imperfect, occupational choice decisions depend on asset holdings and do not maximize the marginal product of time. The occupational choice between working and self-employment is now described by the curve $\overline{R}_1$ in (z_m, z_w) space. Note that a proportional change in both skills decreases the income ratio $\frac{y_{se}}{y_w}$ because the increase in z_m leads to a tighter borrowing constraint for a fixed asset level a, implying that the curve $\overline{R}_1$ tilts down relatively to the ray R_1 as z_m increases. An increase in assets (a) relaxes the borrowing constraint (μ decreases and r_{mw} increases) making it more likely that individuals will choose self-employment so that the position of the $\overline{R}_1$ depends on asset holdings.

We now analyze the decisions of employers. Employers choose how much of their time to allocate to managerial versus working activities and how much capital (k) and (outside) labor services (n_d) to use in production. Using the linear homogeneity of the production function the income of an employer with a units of assets can be written as

$$y_e = MPt_m\, t_m + MPt_w\, t_w + MPn_d\, n_d + MPK\, k + ra - k(r+\delta) - w\, n_d - c_f,$$

where MPt_m and MPt_w denote the marginal products of managerial time and working time, respectively, and MPK and MPn_d represent the marginal product of capital and labor services. We are now ready to prove the following proposition:

Proposition 4 (Employers)

1. *The optimal time devoted to management by employers is* $t_m^* = \min\left\{\hat{t}_m, 1\right\}$, *where* $\hat{t}_m^* \equiv \left[\frac{z_m^\gamma \overline{k}(a, z_m, z_w; \phi)^\nu \theta}{w}\left(\frac{\theta z_w}{\gamma}\right)^{\theta-1}\right]^{\frac{1}{1-\theta-\gamma}}$. *Moreover, denoting by* $k^u(z_m, z_w)$ *the level of capital chosen by an unconstrained entrepreneur with ability* (z_m, z_w), *there exist* $a^*(z_m, z_w) < k^u(z_m, z_w)$ *such that* $t_m^* = 1$ *for all* $a \geq a^*(z_m, z_w)$.

2. *The marginal product of employer's time satisfies:*

$$MPT_e = z_m r_m \geq z_w\, w (\textit{with strict inequality if}\ t_m = 1),$$

where $r_m = \gamma\left[\left(\frac{\nu}{(r+\delta+\mu)}\right)^\nu \left(\frac{\theta}{w}\right)^\theta\right]^{\frac{1}{1-(\nu+\theta)}}$ *is the rate of return to the managerial input* z_m *and* μ *is the Lagrange multiplier associated to the borrowing constraint.*

3. *The income of an employer with ability* (z_m, z_w) *with assets* a *is given by*

$$y_e = z_m r_m + \mu k + ra - c_f,$$

where $k = \overline{k}(z_m, z_w, a)$

Proof. See appendix.

Proposition 4 states that there is a threshold level of asset holdings $a^*(z_m, z_w)$ such that for assets below this level the marginal product of entrepreneurial time is equal to wz_w and the time allocation problem of the employer features an interior solution in which the employer performs both managing and working activities. If asset holdings are higher than the threshold $a^*(z_m, z_w)$, then the marginal product of entrepreneurial time is higher than that as a worker and the time allocation problem exhibits a corner solution $t_m = 1$.

The marginal product of employers' time (MPT_m) can be expressed as the product of managerial skills z_m and the rate of return r_m on the employer's managerial skill. The rate of return (r_m) depends on parameters of the production technology, the real interest rate (r), and the Lagrange multiplier (μ) associated to the borrowing constraint. Note that borrowing constraints (μ) generate heterogeneity in rate of returns to skills among employers.

Proposition 4 shows that when capital markets are perfect ($\mu = 0$) then the marginal product of employer's time is proportional to her managerial ability z_m and the income difference between being an employer and being self-employed $y_e - y_{se}$ is independent of asset holdings. In this case, Proposition 5 shows that the decision of whether to be an employer or to be self-employed only depends on the ability ratio $\frac{z_w}{z_m}$, provided the fixed cost of operation faced by employers is equal to zero ($c_f = 0$). There exist a constant ratio R_2 such that individuals with a skill ratio $\frac{z_w}{z_m}$ below R_2 choose to be an employer. When the fixed cost of operation is positive, then the occupational choice decision depends on comparative advantage (skill ratio $\frac{z_w}{z_m}$) and on the absolute level of managerial ability z_m. Now, to be an employer rather than self-employed the ability vector (z_m, z_w) should satisfy $\frac{z_w}{z_m} < R_2(1 - \frac{c_f}{z_m r_{mw}})^{\frac{\gamma+\theta}{\theta}}$. Intuitively, in the presence of fixed cost of being an employer, employers need a minimum level of managerial ability z_m in order to recoup the fixed cost of operation. The occupational choice decision between employer and self-employment is not only based on the skill ratio.

When capital markets are imperfect and borrowing constraints bind, occupational choice decisions depend on asset holdings because both the marginal product of time and the return to capital of both employed and self-employed individuals depend on their asset holdings (see Proposition 5). Intuitively, an increase in asset holdings increases the employer region in the occupational map in (z_w, z_m) relative to the self-employment ($\overline{R_2}$ in Proposition 5 shifts up). The key is that borrowing constraints tend to be tighter for employers than self-employed since employers need to operate at a larger scale.

Proposition 5 (Employer versus Self-employment) *Let* $R_2 \equiv \left(\frac{r_m}{r_{mw}}\right)^{\frac{\theta+\gamma}{\theta}}$*, where* r_{mw} *and* r_m *are the rate of returns to the skill composite* $\left(z_m^\gamma z_w^\theta\right)^{\frac{1}{\gamma+\theta}}$ *and the managerial skill defined in Propositions 2 and 4, respectively. Then,*

1. *If capital markets are perfect* ($\phi = 1$)*, individuals prefer to become employers relative to self-employment when the ability ratio is such that* $\frac{z_w}{z_m} < R_2(1 - \frac{c_f}{z_m r_{mw}})^{\frac{\theta+\gamma}{\theta}}$.

2. *If capital markets are imperfect ($\phi < 1$), individuals prefer to become employers relative to self-employment when the ability ratio is such that*

$$\frac{z_w}{z_m} < \left[\frac{r_m}{r_{mw}} + \frac{(\mu_e k_e - c_f)}{z_m r_{mw}} - \frac{(\mu_{se} k_{se})}{z_m r_{mw}}\right]^{\frac{\theta+\gamma}{\theta}} \equiv \overline{R_2},$$

where μ_e and μ_{se} are the Lagrange multipliers associated to the borrowing constraints when the individual is an employer or is self-employed, respectively, and k_e and k_{se} are the capital used in production at these occupations.

Proposition 6 collects results characterizing occupational choice decisions when capital markets are perfect ($\phi = 1$). If the fixed cost of operation of employers is $c_f = 0$, occupational choices are only determined by the ability ratio $\frac{z_w}{z_m}$. Depending on parameter values (equilibrium returns to ability), the equilibrium may feature self-employed individuals or not. If equilibrium prices are such that $R_1 > R_2$, then individuals with an ability ratio $\frac{z_w}{z_m} > R_1$ work for a wage, individuals with $R_1 > \frac{z_w}{z_m} > R_2$ are self-employed, and those with $\frac{z_w}{z_m} < R_2$ are employers. A positive fixed cost of operation ($c_f > 0$), implies that employers require a minimum scale in order to operate a profitable business so that the decision to be an employer depends both on the skill ratio $R_1 > \frac{z_w}{z_m} > R_2$ and on the level of managerial ability z_m.

Proposition 6 (Occupational maps when capital markets are perfect ($\phi = 1$) *Assume that $\phi = 1$. Let $R_1 \equiv \left(\frac{r_{mw}}{w}\right)^{\frac{\theta+\gamma}{\gamma}}$ and $R_2 \equiv \left(\frac{r_m}{r_{mw}}\right)^{\frac{\theta+\gamma}{\theta}}$, where r_{mw} and r_m are the rate of returns to the skill composite $\left(z_m^{\gamma} z_w^{\theta}\right)^{\frac{1}{\gamma+\theta}}$ and the managerial skill defined in Propositions 2 and 4, respectively.*

1. *If there are no fixed cost of operation of being an employer ($c_f = 0$), then the optimal occupational choice is the one that maximizes the marginal product of time and is only determined by the skill ratio ($\frac{z_w}{z_m}$) as follows:*

   (a) *If equilibrium prices are such that $R_1 > R_2$, then individuals with an ability ratio $\frac{z_w}{z_m} > R_1$ work for a wage, individuals with $R_1 > \frac{z_w}{z_m} > R_2$ are self-employed, and those with $\frac{z_w}{z_m} < R_2$ are employers.*

   (b) *If equilibrium prices are such that $R_1 < R_2$, there is no self-employed individuals in equilibrium. Individuals with a skill ratio such that $z_w w < z_m r_m$ choose to become employers. Otherwise, they choose to work for a wage.*

2. *If employers incurred a positive fixed cost of operation ($c_f > 0$), the decision to be an employer depends on the skill ratio ($\frac{z_w}{z_m}$) and on the absolute level of managerial ability (z_m). Individuals prefer to become employers relative to self-employment when the ability ratio is such that $\frac{z_w}{z_m} < R_2(1 - \frac{c_f}{z_m r_{mw}})$.*

Summarizing, we have developed a theory with three occupational choices and characterize occupational decisions. The theory implies that, in the absence of capital market imperfections, the skill ratio $\frac{z_w}{z_m}$ drives occupational choices: Workers have a high $\frac{z_w}{z_m}$ ratio, employers a low $\frac{z_w}{z_m}$ ratio, and the self-employed have an intermediate skill ratio. Capital market imperfections distort returns to skill and, thus, occupational choices. A tight borrowing constraint depresses the rate of return to the managerial ability of employers and the return to the composite skill input supplied by self-employed individuals. It also increases the rate of return to capital faced by entrepreneurs. As a result, asset holdings matter importantly for occupational choice decisions in the presence of financial frictions. These results can be illustrated by drawing the occupational maps in our calibrated model economy. Figure 2.10 draws the occupational map when perfect enforcement is introduced in the calibrated model economy. As shown in Proposition 6, when capital markets are perfect occupational choices are determined by the $\frac{z_m}{z_w}$ skill ratio. In our baseline economy, with limited enforcement ($\phi < 1$), occupational choices are determined by the skill ratio and asset holdings since borrowing constraints affect the returns to skills and assets. Figure 2.11 represents graphically, for two fixed asset levels, how occupation varies across individuals that differ on (z_m, z_w). In Panel a, the level of assets is fixed at the median income and in Panel b it is fixed at the mean income. A comparison of the occupational maps, reveal that capital market imperfections expand the region where self-employment is optimal at the expense of the regions where employer and worker are the preferred occupational choices.

2.5 Quantitative Analysis

2.5.1 Calibration

We partition the parameters in the model economy in two. The first group includes the parameters that are set using estimates from other studies in the literature. The second group consists of all the parameters that are calibrated by simulating the model economy.

Parameters set exogenously The model period is set to an year. The international interest rate is set at 3%. The utility function is assumed to be of the CES type:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

with $\sigma = 1.5$. The parameters of the production function are set to standard values in the literature: $\gamma = .2, \nu = .3, \theta = .5$ (see Guner, Ventura, and Xu (2008), Buera, Kaboski, and Shin (2011)). The annual depreciation rate is set to at $\delta = 0.06$.

Calibrated parameters For ease of exposition, below we list the parameters to be calibrated together with a corresponding target that helps identify each parameter. Nonetheless, it is important to keep in mind that the calibration is a multidimensional mapping in which all parameters and calibration targets are inter-related.

1. The discount factor β is chosen so that the capital to income ratio in the steady is equal to 2.4, which is consistent with the capital to income ratio in Brazil (see Júnior, Bugarin, Gomes, and Teixeira (2004)).

2. Enforcement of credit contracts ϕ to match a credit to GDP ratio of 43% in Brazil[5].

3. The coefficients on the quartic polynomio on age determining how the two working and managerial sills vary with age are set so that the age-profile of mean earnings for workers and entrepreneurs are roughly consistent with the data.

4. Following Storesletten, Telmer, and Yaron (2005), the parameters determining the stochastic process on working ability such as the variance of fixed effects $\sigma^2_{\alpha_w}, \sigma^2_{\alpha_m}$, persistence of autoregressive process ρ_w, and the variance of the innovation to working ability over the life-cycle $\sigma^2_{\epsilon_w}$ to match the age profile of the variance of log wages.

5. There are various parameters determining the stochastic process on managerial ability (i) the variance of fixed effect on managerial skills $\sigma^2_{\alpha_m}$; (ii) variance of innovations to managerial abilities $(\sigma^2_{\epsilon_m})$; and (iii) the persistence of the auto-regressive process on managerial ability (ρ_m). To pin down these parameters, we target: (i) the proportion of entrepreneurs and workers in the population of households (32% versus 68%); (ii) the variance of entrepreneurial log-earnings (1.06); and (iii) the persistence of being an employer between two consecutive years (68%).

6. The parameter ρ driving the correlation between managerial and working skills is pinned down by targeting the ratio of median earnings between entrepreneurs and workers.

7. The fixed cost of operation of employers c_f is set to match the fraction of employers among entrepreneurs (one fourth).

Discretization of shocks To solve the model numerically, we first find a finite state approximation of the following bivariate process describing the life-cycle shocks to skills

$$u_t = Au_{t-1} + \epsilon_t,$$

[5]We use the average Private Credit/GDP from 2003 until to 2010 from the World Development Indicators from the World Bank

where u_t is a 2×1 vector, A is a 2×2 matrix, and ϵ is a 2×1 vector with mean 0 and variance-covariance matrix $\Sigma = E(\epsilon\epsilon^{'})$. Using that Σ is a symmetric matrix, we can express it as follows:

$$\Sigma = Q\Lambda Q^{'}, \tag{2.10}$$

where Λ is a diagonal matrix (with the eigenvalues of Σ in the diagonal) and Q is the matrix of eigenvectors of Σ as columns. The bivariate process can be expresed

$$\widehat{u_t} = \widehat{A}\widehat{u}_{t-1} + \widehat{\epsilon_t},$$

where $\widehat{u_t} = Qu_t, \widehat{A} = Q^{'}AQ$, and $\widehat{\epsilon_t} = Q\epsilon_t$. The key to this transformation is that $\widehat{\epsilon_t}$ has a diagonal variance-covariance matrix: $E(\widehat{\epsilon_t}\widehat{\epsilon_t}^{'}) = Q^{'}\Sigma Q = \Lambda$. We then approximate $\widehat{u_t}$ with a Markov chain with states given by a matrix $\widehat{U_t}$ with dimension 2×100. Then the states of the Markov chain which approximate u_t are given by the matrix $U = Q\widehat{U_t}$ with dimension 2×100. Because of our life-cycle environment, the variance of shocks grow with age. To deal with this feature, we allow the support of the shocks and the Markov chain to change with age. The Markov chain is allowed to vary with age so that the finite state approximation of the autoregressive bivariate process matches the unconditional variance of the continuous bivariate shock process at each age.

Regarding fixed effects, the bivariate normal distribution is discretized with 3 values for working skill and 5 values for managerial skills. As a result, there are 15 pairs of fixed effects. At each age, there are 1500 possible pairs of skills (z_w, z_m).

2.5.2 Calibration results

We now discuss how the calibrated model economy matches the calibration targets. The values of the calibrated parameters are reported in Table 2.1.

Table 2.2 shows that the model economy matches reasonably well the targets for the credit to GDP ratio of 43% and the capital to income ratio of 2.4. Figure 2.5 compares the variance of log-earnings of workers in the model economy with the Brazilian data. The model economy is consistent with the fact that there is a large amount of inequality early in the life cycle and that inequality grows substantially with age over the life cycle. The stochastic process on working skills is characterized by a high persistence (ρ_w close to 1), which is needed to match the linear age-profile of the variance of log wages in the Brazilian data. This is consistent with the findings of Storesletten, Telmer, and Yaron (2005) for the US. Relatively to previous findings for the US economy, the calibration requires a large variance of individuals fixed effects (σ_{α_w}) to match the high inequality of wages at age 20 in Brazil.

The calibration implies that the variances of fixed effects and of the innovation of managerial skills are much larger than the corresponding variances of working skills (see Table 2.1). This is necessary for the model economy to be consistent with the large variance of entrepreneurial earnings in the Brazilian data.

Table 2.3 compares the fraction of households that are workers, self-employed, and employers in the calibrated model economy and in Brazil. The model economy matches quite closely the fractions of workers (68%), self-employed (24%), and employers (8%) in the data. The calibration also matches the fact that about 70% of the employers in Brazil at a given point in time are still employers one year after (see Table 2.3). In the calibrated model economy the persistence of entrepreneurial shocks is high ($\rho_m = 0.78$), but less than the persistence of shocks on working ability ($\rho_w = 0.98$)

The calibration implies that the correlation between skills is positive but moderate ($\rho = 0.1$). This is necessary for the economy to be consistent with the fact that ratio of median income between entrepreneurs and workers is equal to 1 in the Brazilian data. In a sensitivity analysis, we later calibrate two economies in which we exogenously set a high and low skill correlation. We find that a high skill correlation implies a counterfactually high ratio of median income between entrepreneurs and workers. Conversely, a calibration with a negative skill correlation implies that the median income of entrepreneurs are below that of workers, which is also counterfactual.

Summing up, we believe that the calibrated model economy provides a reasonable account of income inequality and occupational choices in Brazil.

2.5.3 Performance of baseline economy.

We now discuss how the baseline economy matches some facts on occupational transitions and on the distribution of income across occupations that were not directly targeted in the calibration.

Table 2.4 reports predictions of the model economies on occupational transitions between the three occupations considered (worker, self-employed, employer). While there are 9 possible occupational transitions, we remind the reader that the calibration only targeted the persistence of being an employer between two consecutive years. The baseline economy matches the patterns on the persistence of occupational choices remarkably well (see Table 2.4). First, consistently with the data, the model economies predict that being a worker is quite persistent: 90% of workers in the model economy are workers one year later. This percentage is about 94% in the data. Both in the baseline economy and in the data, entrepreneurs are less likely to remain in their occupation than workers. Second, examining transition rates within the entrepreneurial class, the calibrated model economy matches the fact that individuals are much more likely to transit into employer from self-employment than from being a paid worker. In the data, individuals are about three times more likely to become employers if they are self-employed rather than working for a wage. Indeed, the (annual) transition rate from self-employment into employer is 22% while the transition rate from worker to employer is only 8%. In the model these rates are 26% and 8%, respectively. Third, the (annual) transition rates out of employer in the data implies that employers are much more likely to switch to self-employment (8%) than to paid work (1%). In the model economy, these transition rates are 6% and 1%, respectively.

The model economy was calibrated to match two statistics on the distribution of income across workers and entrepreneurs (ration of median income and the variance of income of each of the two occupations). Figure 2.6 plots the distribution of income across these two occupation categories in the data and in the model. The model was calibrated to match the fact that the ratio of median income between entrepreneurs and workers is equal to one and that the variance of entrepreneurial income is higher than that of workers. Figure 2.7 plots the distribution of income across three occupations (e.g. the entrepreneurial category is subdivides in two groups: employers and self-employed). The baseline economy is consistent with the fact that there is substantial income heterogeneity in all three occupations. Moreover, as in the data, the distribution of self-employment income is shifted to the left relative to that of workers and the distribution of worker's income is shifted to the left relative to that of employers. Nonetheless, we emphasize that there is substantial heterogeneity among employers: Some low income employers make less income than the median self-employed individual.

All in all, the model economy matches reasonable well patterns on occupational transitions and distribution of income within and across occupations. While we could have built a theory with three distinct skills (one for each occupation) to better match the facts, it is remarkable that our theory with heterogeneity in two skills provides an excellent account of the patterns in the data on the transition rates into and out of the employer occupation. Below, when performing a sensitivity analysis, we evaluate other implications of the theory.

2.5.4 Experiment: Removing financial frictions.

In order to assess the effects of credit market institutions, we compute equilibrium in the baseline economy under the assumption of perfect credit-enforcement institutions ($\phi = 1$). We focus on how capital markets impact on occupational choices, resource allocation, aggregate output, and the distribution of income in the economy.

2.5.4.1 Occupational structure and financial frictions.

We find that removing financial frictions has important consequences on the occupational structure in the economy. Indeed, the fraction of entrepreneurs decreases from 33% to 18% (see Table

2.5). Moreover, most of the decrease in entrepreneurship is due to a large decrease in the rate of self-employment: While self-employment rates drop from 24% to 11%, the fraction of employers drops by about 2 percentage points. Altogether, the theory is consistent with key stylized facts on changes in the occupational structure with economic development. Consistently with the data, the theory that the changes in rates of entrepreneurship across rich and poor countries is due to changes in the self-employment rate. The theory is also consistent with the fact that the fraction of workers in the labor force tends to increase with economic development: It increases from 68% in the baseline economy to 82% with perfect capital markets.

The mechanism through which capital markets impact on the occupational structure is through its impact on equilibrium wages. When financial frictions are removed, the wage rate increases due to a better allocation of productive resources (employers are better selected and they can use more capital). The increase in the wage rate strongly discourages self-employment, as can be seen by comparing the self-employment region (green region) in the occupational maps drawn in Figure 2.10 and Figure 2.11. Note that capital market imperfections expand the region where self-employment is optimal at the expense of the regions where employer and worker are the preferred occupational choices.

2.5.4.2 Output, resource allocation, and financial frictions.

The gain in aggregate output of eliminating financial frictions is quite large: It amounts to an increase of 48% (see Table 2.6). There is also a substantial change in the sectorial composition of output. While production by employers increase by 64%, production by self-employed decreases by 53%. This should not be surprising given that the removal of financial frictions leads to large reduction in the rates of self-employment in the baseline economy.

We now consider the impact of financial frictions on the efficiency of production. To this end, we compute (TFP) as the (input-weighted) average productivity with which the composite capital and labor input is used in production across entrepreneurs

$$TFP = \int_{i \in E} m_i^{\gamma} \frac{k_i^{\nu} n_i^{\theta}}{\int_{i \in E} k_i^{\nu} n^{\theta} di} di,$$

where $m_i = (z_{mi} t_i)^{\gamma}$ is the managerial input used by entrepreneur i and similarly for n_i and k_i. We find that aggregate TFP increases by 24% when financial frictions are removed. TFP increased by 9% and 16% among self-employed and employers, respectively. Note that the aggregate TFP gain (24%) is higher than the gain in each of the two sectors (9% and 16%). The reason is that the removal of financial frictions leads to a reallocation of capital from self-employed to employers: The share of aggregate capital used by the self-employed drops from 19% to 4%. Because employers tend to have a higher managerial ability than self-employed entrepreneurs, the reallocation of capital between these two groups enhances the aggregate productivity gains of removing financial frictions.

Having said that self-employed are less productive entrepreneurs than employers, it is important to recognize that self-employment reduces the impact of financial frictions on output. To assess how self-employment affects the impact of financial frictions on aggregate output, we shut down self-employment in the baseline economy and simulate the removal of financial frictions. We find that the output gain is 53%, which is higher than the 48% increase obtained when simulating the removal of financial frictions in the baseline economy. Hence, self-employment decreases the negative impact of financial frictions on aggregate output. This finding can be explained as follows: Financial frictions make it hard for young and talented entrepreneurs (individuals with high managerial skills) to raise external funds. This effect is compounded by the fact that financial frictions depress the equilibrium real wage, which makes it difficult to accumulate savings by working for a paid wage when young. Self-employment allows talented entrepreneurs to circumvent the low wage and build up savings, diminishing the negative impact of financial frictions on aggregate output.

It is also interesting to decompose the gains in TFP due to the removal of financial frictions between the ones that are due to a better allocation of capital and labor inputs across entrepreneurs (intensive margin) versus the gains coming from a better selection of entrepreneurs (extensive margin). To this end, we re-distribute the capital in the baseline economy in order to equate the marginal product of capital across all entrepreneurs.[6] We find that capital reallocation leads to a TFP gain of 1.3% for the self-employed and of 12% for employers (see Table 2.7). Hence, reallocation of capital accounts for 14% of the gains in among the self-employed and for 76% of the gains among employers. At the aggregate level, reallocation accounts for about half of the TFP gains (54%).

The result that the intensive margin accounts for a much larger fraction of the TFP gains of employers than of the self-employed, suggests that borrowing constraints are tight among the former but not among the latter. Figure 2.8 plots the histogram of the ratio of the marginal product of capital relative to the gross interest rate $(r + \delta)$ across employers and self-employed individuals. Note that this ratio is equal to one for about 85% of self-employed, indicating that the vast majority of the self-employed are not borrowing constraint. The reason is that self-employed individuals tend to operate their businesses at a much smaller scale than employers (their mean managerial input is about one-twentieth the one of employers) and do not need to borrow much capital. On the other hand,the marginal product of capital varies substantially across employers and the vast majority of them face a binding borrowing constraint.

[6]While the marginal product of the labor input is equated across employers, this equality does not hold across self-employed individuals because we assume that labor input is not movable across the self-employed. Alternatively, we could have allowed self-employed to pay a fixed cost in order to equate the marginal product of labor. However, this would effectively imply an occupational switch (from self-employer to employer) which we consider as part of the extensive margin.

2.5.4.3 Impact of financial frictions on the distribution of income.

Financial frictions affect the distribution of income in many ways. While the overall effect on income inequality is not large[7], it has important effects on the sources of income inequality and on its persistence over time.

We start by analyzing how financial frictions affect the distribution of capital versus non-labor income. Capital income is computed as $ra + \mu k$. Non-capital income is computed as the sum of labor income, managerial rents, and self-employment rents. Abusing terminology from now on we refer to non-capital income as labor income. Table 2.14 compares the Gini indexes of capital and labor income across economies. We find that capital income is much more unevenly distributed than labor income both in the baseline economy and in the perfect capital market economies. Surprisingly, we find that capital market imperfections have opposing effects on the concentration of labor and capital income. Labor income is more evenly distributed in the baseline economy than in the economy with perfect enforcement, with a Gini index of .52 in the former economy and of 56 in the latter economy). On the other hand, the Gini index of capital income is about 10 percentage points higher in the baseline economy. The opposite effects of capital market imperfections on the distributions of capital income and labor income offset each other and account for the small change in the Gini index of income.

The fact that the distribution of factor income varies so much across economies is symptomatic of the resource misallocation prevalent under imperfect capital markets. The low concentration of the distribution of labor income in the baseline economy is due to the fact that borrowing constraints distorts rate of returns to managerial ability (recall that $\mu > 0$ reduces r_m). Moreover, in the baseline economy returns to managerial ability r_m and managerial ability z_m are strongly negative correlated, with a correlation coefficient of -0.5. Thus, skillful managers tend to receive low returns to their ability. On the other hand, the correlation coefficient between these two variables is zero in the economy with perfect capital markets, as there is no heterogeneity in rate of returns to ability.

Capital income is highly unequal in the baseline economy because there is substantial heterogeneity in the returns to capital. The interest rate on deposits (3%) is substantially smaller than the average marginal product on capital obtained by employers (13.2% net of depreciation). Moreover, the marginal product of capital across employers varies importantly and its distribution features a coefficient of variation above .60. Again, this fact is symptomatic of resource being inefficiently allocated.

The presence of borrowing constraints imply that the returns to managerial ability are positively correlated with capital income. Hence, the correlation between capital and labor income is equal to .80 in the baseline economy, which is much larger than the .50 value in the economy with

[7]The Gini index of income decreases from 0.53 to 0.52 with the removal of financial frictions

perfect capital markets. In the latter, the positive correlation between capital and labor income is due to the fact that highly able people tend to hold more capital than low ability people but not to rate of return differentials.

2.5.4.4 Capital markets and the persistence of income.

To evaluate the effect of imperfect capital markets on the persistence of income, we simulate data in the baseline economy and in the economy with perfect capital markets and run the following regression in

$$log(y_{t,j}) = \alpha_j + \beta log(y_{t-1,j}) + b_2 age_t + b_3 age_t^2, \tag{2.11}$$

where y_{tj} represents the income of individual j at age t, α_j is an individual fixed effect, and β measures the persistence of log-income. We find that removing financial frictions in the baseline economy reduces the estimated value of β from 0.81 to 0.74. Income is more persistent in the baseline economy because assets are positively correlated with rate of returns and because assets matter for occupational choices. On the other hand, when there is perfect enforcement assets do not affect rates of returns and occupational choices and the persistence of income is only driven by the persistence of shocks and asset holdings.

2.5.4.5 Capital markets and the distribution of consumption.

Financial frictions have an heterogeneous impact across households. To assess the distributive impact of financial frictions, Table 2.15 compares consumption inequality in the baseline model economies with that in an economy with perfect enforcement of credit contracts $\phi = 1$. We find that financial frictions have important effects on the distribution of consumption. We find that the Gini coefficient of consumption is 2 percentage points lower in the economy with perfect enforcement of credit contracts ($\phi = 1$). It is interesting that financial frictions have opposite effects on the inequality at the top and the bottom of the consumption distribution. The ratio of consumption between the 10th percentile and the 50th percentile of the consumption distribution is equal to 0.29 in the baseline model economy. This ratio increases to 0.33 in the economy with $\phi = 1$. On the other hand, the consumption ratio between households at the 90th and 50th percentile increases from 3.3 to 3.44. Hence, relative to the perfect credit economy, the baseline model economy has more inequality at the bottom of the consumption distribution but less inequality at the top. The first effect is more important than the latter effect so that overall consumption inequality, as measured by the Gini index, is higher in the baseline economy than in the $\phi = 1$ economy.

2.5.5 Is the skill distribution important for the aggregate effects of financial frictions?

To show that the skill distribution matters importantly for the impact of financial frictions we consider two new economies in which we fixed exogenously the skill correlation. In one economy, the skill correlation is fixed to a high positive value ($\rho = 0.8$ and in the other to a high negative value ($\rho = -0.8$). The two economies are re-calibrated to the same targets of the baseline economy, except for the fact that we do not target the ratio of median earnings between entrepreneurs and workers. We then compare across economies the impact of removing financial frictions.

2.5.5.1 The impact of financial frictions across economies.

The output gains of removing financial frictions vary substantially across the three calibrated model economies (see Table 2.10).[8] The output gains range from 36% to 55% as the correlation between skills decreases from 0.8 to -0.8. Similarly, the TFP gains across these economies range from 22% to 31%. Hence, financial frictions have much lower effects on output per worker and TFP when skills are (strongly) positively correlated. The correlation between skills matters for the impact of financial frictions because it shapes the correlation between savings and managerial talent in equilibrium. In particular, the financing problems faced by talented entrepreneurs are less severe when entrepreneurs are also talented workers. When individuals are highly skillful both at managing and at working, they can work for a wage and build savings in order to diminish the negative effects of borrowing constraints on their entrepreneurial income. On the other hand, talented entrepreneurs find it more difficult to accumulate savings when skills are negatively correlated, making borrowing constraints tighter. Moreover, talented entrepreneurs have a harder time building up savings relative to other individuals with lower entrepreneurial talent because the latter have higher working skills. By allowing individuals with low managerial talent to build savings faster, financial frictions provide low skilled managers with a comparative advantage at entrepreneurship. These effects explain why the skill correlation matters importantly for the output and TFP losses caused by financial frictions.

A key innovation of our theory is that it allows entrepreneurs to choose what fraction of their time they allocate to managing versus working. This assumption allows our theory to be consistent with the fact that self-employment is quite important in poor countries. We now show that self-employment matters for the quantitative impact of financial frictions in the three calibrated model economies. To this end, we shut down self-employment in all economies and simulate the impact of financial frictions. We find that in all the calibrated model economies the output

[8] The match of the calibrated targets is better in the baseline economy. The calibrated parameters and calibration results for the economies with very positive or very negative correlation are available upon request.

gains due to the elimination of financial frictions increase substantially in the absence of self-employment (by between 6 to 8 percentage points). Since wages are low with financial frictions, self-employment allows individuals with high managerial skill to attain higher earnings. This effect is most important in the economy with negative correlation of skills since in this economy individuals with high managerial ability tend to have lower working ability.

To further understand how the skill correlation matters for the impact of financial frictions, we analyze how financial frictions distort the rate of returns to the various production inputs. Recall that when capital market are perfect the rate of return of all productive inputs are equalized across production units. However, rates of return do vary across production units under financial frictions (see Section 4). We now show that the skill correlation parameter ρ matters importantly for the variation in rate of returns caused by financial frictions. Table 2.12 compares the variation in rates of returns among employers and self-employed individuals in the calibrated model economies. The standard deviation of the marginal product of capital among employers is twice as large in the economy with $\rho = -0.8$ than in the economy with $\rho = 0.8$ (.14 versus .07). The variation in rates of returns to capital reflects the variation in the tightness of the borrowing constraint across entrepreneurs.[9] The results in Table 2.12 show that when skills are positively correlated there is less heterogeneity in rates of return on capital across entrepreneurs than when skills are negatively correlated. Intuitively, the financing problems faced by talented entrepreneurs are less severe when entrepreneurs are also talented workers. This is because households with high working skills can rapidly accumulate savings and alleviate the financial constraints that limit the operation of their businesses. On the other hand, when skills are negatively correlated borrowing constraints are tighter because talented entrepreneurs find it more difficult to accumulate savings.

Borrowing constraints also generate heterogeneity on the rate of return to the managerial input among employers (r_m) and on the rate of return on the self-employment composite(r_{mw}). Both of these returns decrease with the tightness of financial constraints (see Section 4). Table 2.12 shows that the variation in rates of return to the managerial input among employers is about twice as large in the economy with $\rho = -0.8$ than in the economy with $\rho = 0.8$ (0.31 versus 0.17.) Moreover, while in all economies the return to the managerial skill is negatively correlated with the level of managerial ability, this correlation is the lowest in the economy with $\rho = -0.8$ (about -0.63). When skills are strongly negatively correlated, the tight borrowing constraints faced by entrepreneurs with high managerial skills imply that they obtain a lower return to their skills than less able entrepreneurs. In this case financial frictions generate a strong comparative advantage at entrepreneurship for households with lower managerial talent but higher working ability, reducing the average entrepreneurial ability, and total factor productivity. Table 2.9 shows that changes in TFP associated with the elimination of financial frictions range from

[9]Recall that the marginal product of capital can be expressed as $MPK = r + \delta + \mu$, where μ represents the Lagrange multiplier associated to the borrowing constraint (see Section 4).

31% to 22%, with the largest (lowest) increase attained in the economy with strongly negative (positive) correlation of skills.

2.5.5.2 Comparing economies with different correlation of skills

We have shown that the impact of financial frictions vary substantially across the three calibrated model economies. Then, in order to assess the impact of financial frictions in the Brazilian economy it is important to use Brazilian data to test the predictions of the calibrated model economies.

Skill correlation and distribution of income across occupations. Figure 2.12 shows that the calibrated model economies differ importantly in the distribution of income by occupation (workers versus entrepreneurs). The economy with strongly correlated shocks $\rho = 0.80$ is grossly at odd with the brazilian data: It counterfactually predicts that the distribution of earnings of workers is shifted to the left relative to that of entrepreneurs. The economy with strongly negative correlated shocks $\rho = -0.80$ is also at odds with the brazilian data since it implies that the distribution of earnings of workers is shifted to the right relative to that of entrepreneurs. On the other hand, our baseline economy (with $\rho = 0.10$) fits the brazilian evidence on the income distribution across occupations reasonably well. Indeed, the baseline economy was calibrated to match the ratio of median income between workers and entrepreneurs (which is 1.0 both in Brazil and in the baseline model economy). This statistic takes a value of 0.7 in the economy with $\rho = 0.8$ and a value of 1.3 when $\rho = -0.8$.

Figure 2.13 compares the distribution of income in the calibrated model economies and Brazil when the population is divided in three occupational groups (workers, self-employed, and employers). The economy with strongly correlated shocks counterfactually predicts that the earnings distribution of self-employed individuals is shifted to the right relative to that of workers. As ρ decreases, the distribution of earnings of self-employed individuals shifts to the left. As a result, consistently with the evidence, the economies with $\rho = 0.10$ and -0.80 exhibit a distribution of earnings of self-employed households that is shifted to the left relative to that of workers. Overall, the economy with $\rho = .10$ is the one that fits the evidence best. Relative to the data, the economy with a strong negative skill correlation implies that the self-employed individuals have too low earnings relative to workers.

We now discuss why the correlation between skills matters importantly for the income distribution across occupations. When ρ is sufficiently high, the correlation between $ln(z_{mt}/z_{wt})$ and $ln(z_{wt})$ becomes positive. In this case, a high skill ratio z_{mt}/z_{wt} is also associated with high values of z_{mt} and z_{wt} so that households that have a *comparative advantage* at managing (high z_{mt}/z_{wt}) also have an *absolute advantage* in both skills. When entrepreneurs have an absolute advantage in both occupations, highly skilled workers tend to have a comparative advantage at managing and choose the entrepreneurial occupation. Low skill workers do not have a comparative advantage at managing and choose to work for a wage. As a result, the earnings distribution among entrepreneurs is shifted to the right relative to the earnings distribution among workers and earnings inequality between occupations is large. On the other hand, when the correlation between $ln(z_{mt}/z_{wt})$ and $ln(z_{wt})$ is negative households in one occupation tend to be better at that occupation than households choosing the other occupation. Earnings inequality across occupations is not as large as in the absolute advantage case.

It is easy to show that the skill ratio and the working skill are jointly log-normally distributed for each age t:

$$\begin{pmatrix} ln(z_{mt}/z_{wt}) \\ ln(z_{wt}) \end{pmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{mt}^2 + \sigma_{wt}^2 - 2\rho_{wmt}\sigma_{wt}\sigma_{mt} & \rho_{wmt}\sigma_{wt}\sigma_{mt} - \sigma_{wt}^2 \\ \rho_{wmt}\sigma_{wt}\sigma_{mt} - \sigma_{wt}^2 & \sigma_{wt}^2 \end{bmatrix} \right)$$

The absolute advantage case arises when the correlation between $ln(z_{mt}/z_{wt})$ and $ln(z_{wt})$ is positive, which holds if and only if

$$\rho_{wmt} > \frac{\sigma_{wt}}{\sigma_{mt}}$$

Thus, the correlation of skills have to be sufficiently strong for the absolute advantage case to hold. Figure 2.9 graphs the correlation between $ln(z_{mt}/z_{wt})$ and $ln(z_{wt})$ for the three calibrated model economies. The economy with $\rho = 0.80$ is the only one with a positive correlation. In this economy, households with high managerial ability tend to have an absolute advantage in skills (have higher managerial and working skills). The economy with $\rho = 0.10$ exhibits a correlation between $ln(z_{mt}/z_{wt})$ and $ln(z_{wt})$ of roughly -0.30.

Skill correlation and the persistence of earnings. The correlation between skills also matter for the persistence of earnings over time. Intuitively, earnings are less volatile when skills are positively correlated than negatively correlated. To compare the persistence of earnings across the calibrated model economies, for each economy we simulate artificial data and run the regression in (2.11). We find that the persistent of log-income increases from 0.73 to 0.84 as ρ increases from -0.8 to 0.8. Unfortunately, we do not have panel data from Brazil to estimate the persistence of income in Brazil. Nonetheless, we can use consumption data from Brazilian households to test the predictions of the theory. The idea is that consumption theory implies that permanent income is a key determinant of consumption decisions. Hence, the higher the persistence of income the higher should be the cross-sectional correlation between consumption and income across households (e.g. the correlation between consumption and income at a given date t). Table 2.13 shows that the correlation between consumption and income varies widely across the calibrated model economies: from 0.24 when $\rho = -0.8$, to .79 when $\rho = 0.1$, and up to 0.85 when $\rho = 0.8$. This correlation is 0.71 in the Brazilian data.

Skill correlation and selection of entrepreneurs. The skill correlation matters importantly for how entrepreneurs are selected from the wage distribution. This is shown by comparing wages at a given point in time between those who become entrepreneurs in the following period relative to those who did not. In particular, this is done by running the following regression on log wages:

$$log(y_{t-1}) = b_0 + b_1 * age + b_2 age^2 + b_3 * entrepreneur_t,$$

where y_{t-1} denotes wage income in period $t-1$ and $entrepreneur_t$ is a dummy that takes value of 1 if the individual becomes an entrepreneur in period t. This regression is run for the three calibrated model economies as well as for the Brazilian data.[10]

We find that in the Brazilian data, entrepreneurs are drawn negatively from the wage distribution (the coefficient b_3 is negative and equal to -0.13). The baseline economy also implies a negative coefficient with a value of -0.36 (see Table 2.11). The economy with $\rho = 0.8$ a positive value of b_3. The economy with strongly negative correlated skills implies a too low value for b_3 (-0.86).

To investigate whether self-employed are differently selected from the wage distribution relative to employers, we run the wage regression above but allowing for a dummy for self-employment. We then consider another specification in which we allow for a dummy to indicate those workers who switch into employer. We find that in the Brazilian data self-employed tend to be negatively selected from the wage distribution ($b_3 = -0.24$) but that employers tend to be positively selected ($b_3 = 0.43$). The baseline economy ($\rho = 0.1$) is consistent with these patters (the dummy for self-employment is -0.43 and the one for employers is 0.50).

Altogether, the Brazilian evidence reviewed supports the economy with a moderately positive correlation between managerial and working skills ($\rho = 0.1$).

2.5.6 The political economy of financial frictions

While it is well understood that financial frictions can have a large negative impact on aggregate output and total factor productivity, it is less clear why the institutions leading to poor property rights and contract enforcement are so persistent. In this section we use our theory to gain some insights into the political economy of financial frictions. In our theory, financial frictions have an heterogeneous impact across individuals that differ on age, wealth, and skills. Since our model economy was calibrated to Brazilian household data, we use our theory to assess the distribution of welfare gains of eliminating financial frictions in the Brazilian economy.

We assume that the baseline economy ($\rho = 0.10$) is in steady state and that suddenly and unexpectedly there is a once and for all institutional reform that increases ϕ to 1. On impact,

[10]While the Brazilian data is not a panel, we have data on the occupational choices of individuals one year apart.

the wealth distribution does not matter for occupational choice decisions. Now, the skill ratio $\frac{z_m}{z_w}$ is the only determinant of occupational choice decisions. Workers who could not operate as entrepreneurs because of binding borrowing constraints can now start a business. Moreover, entrepreneurs who were initially borrowing constraint see their managerial rents ($r_m z_m$) and entrepreneurial income increase (see the discussion in Section 4). The importance of this effect varies across individuals: It is more important for talented entrepreneurs (high z_m) because, ceteris paribus, they were more likely to be initially constrained. As talented entrepreneurs raise their demand of capital relative to less talented entrepreneurs, capital is reallocated towards more productive entrepreneurs, increasing the demand for labor, and the equilibrium wage rate. The rise in wages decrease the profits of entrepreneurs. Hence, employers' income can go up or down depending on whether the increase in managerial rents is higher or lower than the increase in labor costs.

The financial reform has non-trivial effects on the distribution of income. Occupational choices of individuals are crucial for understanding how the reform impacts on them. Keeping fixed the occupational choices of the initial steady state, workers gain with the reform since the wage rate increase. Self-employed also gain since they can borrow more. Unconstrained employers loose since wage payments go up but their managerial rents do not increase. Constrained employers may gain or loose depending on whether the increase in managerial rents outweighs the increase in wages. Since the increase in managerial rents is likely to be more important for talented entrepreneurs, these entrepreneurs are more likely to see their income increase. The untalented entrepreneurs who were operating businesses prior to the reform are likely to see their income go down and to switch occupations.

We now explain how we compute the distribution of welfare gains from the reform. The small open economy assumption simplifies the computation significantly. Once the reform takes place, the marginal product of capital will be equated across entrepreneurs and will be equal to the international interest rate plus the depreciation rate of capital. On impact, competition for workers will drive the wage rate to its new long run value, which increases on impact by about 40%. While the distribution of wealth, consumption, and income may change for some periods after the reform, all macroeconomic aggregates (capital, GDP, wage rate) will be constant after the initial period of the reform. Since there are no transitional dynamics in the macroeconomic aggregates, we can then compute the distribution of welfare gains for all individuals alive at the moment of the reform as follows:

1. Simulate the distribution of households across states $s = (age, assets, z_m, z_w)$ from the initial steady state prior to the reform.
2. For each household in state s, compute the permanent consumption compensation in the original steady state that will let the household attain the same utility as in the perfect

credit economy. Denoting by $V^{baseline}(s)$ the discounted lifetime utility of a household in the baseline economy, and $V^{\phi=1}(s)$ the value function in the perfect enforcement economy, the consumption compensation $\lambda(s)$ is computed as follows:

$$\lambda(s) = \left(\frac{V^{\phi=1}(s)}{V^{baseline}(s)}\right)^{\frac{1}{1-\sigma_c}} - 1,$$

where σ_c denotes the curvature of the period utility function in consumption ($\sigma_c = 1.5$). Households with $\lambda(s) > 0$ gain from the elimination of enforcement problems. Households with $\lambda(s) < 0$ see their welfare decrease with the reform of financial market institutions.

We find that the average welfare gain among households alive at the period of the institutional reform is 16.5%. The standard deviation of the distribution of welfare gains is 13.5%. Figure 2.14 shows the distribution of welfare gains across the population and documents that there is substantial heterogeneity. While the vast majority of households gain from the reform, about 8.7% of the population see their welfare decrease with the reform. Who are the households that lose with the reform?

Figure 2.15, 2.16, 2.17, 2.18, show the age, asset, and managerial-skill distributions among those who oppose and support the reform. We find that households that lose from the reform tend to be older, richer, and exhibit higher managerial skills and lower working skills than households that support the reform. These findings are just reflecting that occupational choices are crucial for understanding the political economy of the reform: Among the households that are worse off with the reform, about 93% of them would have been entrepreneurs on the period of the reform had the reform not taken place, and 66% would have been employers.

Employers are a positive selection from the population distribution of managerial skills. Then , the fact that about two thirds of those who oppose the reform are employers explains why the managerial ability of those supporting the reform is higher than that of those opposing the reform. Nonetheless, not all employers support the reform: About 36% of employers in the initial equilibrium benefit from the elimination of enforcement problems. We find that the employers benefiting from the reform tend to be of higher managerial ability than those who oppose it. The reason is that high ability employers are more likely to be borrowing constrained than low ability entrepreneurs. As a result, they are more likely to operate at an inefficient scale and to gain more from the elimination of enforcement problems. On the other hand, the financial reform hurts many of the lower skill employers and force them to change their occupation status: About 46% of the entrepreneurs that oppose to the reform and would have been employers had the reform not taken place, do not hire any labor after the reform (most of them become self-employed after the reform). The wage hike after the reform makes it unprofitable for these entrepreneurs to hire outside labor.

Summing up, while most households benefits from a reform that eliminates enforcement problems, the majority of employers (about two thirds) lose from the reform. By depressing the demand for labor, limited enforcement depresses the equilibrium wage rate, increasing the profits of employers. Obviously, entrepreneurs as a group will benefit even more by forming a cartel in order to restrict labor demand and depress the wage rate. This achieves the goal of depressing the wage rate but without distorting the capital markets. However, this cartel agreement is not incentive compatible as each entrepreneur will have incentives to violate the group agreement and hire labor. Importantly, limited enforcement is an incentive feasible mechanism that leads to a depress wage rate. Our theory thus suggests that employers may have a vested interested in maintaining a status quo with low enforcement.

2.6 Conclusions

We develop a theory of entrepreneurship, financial frictions, and economic inequality in Brazil. The key innovation of our theory is to extend the Lucas (1978) model in order to incorporate heterogeneity in two skills: − working and managerial skills. This modeling feature is necessary for the theory to be consistent with evidence on the income distribution across occupations in Brazil. Moreover, the theory has the novel implication that some entrepreneurs will not hire any outside labor and be own account workers (or self-employed entrepreneurs), which is consistent with the cross-country evidence that the high rates of entrepreneurship in poor countries, such as in Brazil, is mostly due to the prevalence of self-employed workers. We show that financial frictions have a non-trivial impact on inequality: On the one hand, they lead to higher and persistent inequality by generating variation in returns to skills and by making these returns depend on asset holdings. On the other hand, since borrowing constraints tend to be tighter for highly skill than for low skill individuals, financial frictions reduce inequality by diminishing the rents obtained by highly skilled individuals relative to a situation with perfect capital markets.

The model economy is used to simulate the effects of removing financial frictions in Brazil. We find that the changes on the occupational structure are large: The fraction of entrepreneurs decreases from 33% to 18%, with the bulk of the decrease in entrepreneurship explained by the large decrease in the rate of self-employment. The gains in aggregate output of removing financial frictions amounts to 48%, with asymmetric effects in the sectorial composition of output: While production of employers increase by 64%, production of the self-employed decreases by 53%. We also simulate the impact of removing financial frictions when self-employment is shut down in the baseline economy. We find that the output gain is 53%, which is higher than the 48% increase obtained in the baseline economy. Hence, self-employment decreases the negative impact of financial frictions on aggregate output. We find that capital market imperfections have opposing effects on the concentration of labor income and capital income. Labor income is more evenly distributed in the economy with imperfect capital markets than in the economy with no financial frictions (with a Gini index of .52 versus a Gini index .56 in the latter economy). This is because financial frictions depress the rents earn by highly able entrepreneurs relative to an economy with perfect capital markets. On the other hand, the Gini index of capital income is about 10 percentage points higher in the economy with imperfect credit markets.

We evaluate the distribution of welfare gains and losses of eliminating financial frictions. While the vast majority of households gain from a reform that eliminates capital market imperfections, about 8.7% of the population see their welfare decrease with the reform. Households that lose from the reform tend to be older, richer, and exhibit higher managerial skills and lower working skills than households that support the reform. These findings are just reflecting that occupational choices are crucial for understanding the political economy of the reform: Among the households that are worse off with the reform, about 93% of them would have been entrepreneurs on the period of the reform had the reform not taken place, and 66% would have been employers. Our theory thus suggests that employers in Brazil may have a vested interested in maintaining a status quo with low enforcement.

Table 2.1: Calibrated Parameters

	$\rho_{w,m} = 0.1$
ρ_w	0.98
ρ_m	0.78
$\sigma^2_{\alpha,w}$	0.38
$\sigma^2_{\alpha,m}$	1.59
σ^2_w	0.03
σ^2_m	0.99
c_f	0.10
ϕ	0.23
β	0.995

Table 2.2: Calibration Results-Model Aggregates

	Data	Model
K/Y	2.4	2.3
Credit/GDP	43%	42%
Var Log(Earn)-Entrepreneurs	1.1	1.2

Table 2.3: Calibration Results-Occupational Structure

Fraction	Data	Model
Workers	68%	67%
Self-Employed	24%	24%
Employers	8%	9%
Emp to Emp	70%	68%

Table 2.4: Performance of the Model-Transitions

Transitions	Data	Model
W to W	94%	90%
SE to W	5%	9%
E to W	1%	1%
W to SE	14%	28%
SE to SE	78%	64%
E to SE	8%	8%
W to E	8%	6%
SE to E	22%	26%
E to E	70%	68%

In the table above we use W for Workers, SE for Self-Employed and E for Employers.

Table 2.5: Occupational Structure and Financial Frictions

Occupation	Baseline Economy	No Frictions
Workers	67%	82%
Self-Employed	24%	11%
Employers	9%	7%

Table 2.6: Aggregate Effects of Removing Financial Frictions

Changes in %	Output	TFP
Aggregate	48	24
Self-Employed	-53	9
Employers	64	16

Table 2.7: TFP and Financial Frictions

Changes in %	TFP	$TFP_{Reallocation}$
Aggregate	24	13 (54%)
Self-Employed	9	1.3 (14%)
Employers	16	12 (76%)

Table 2.8: Income Inequality and Financial Frictions

	$\phi = 0.23$	$\phi = 1$
Gini Labor Income	0.52	0.56
Gini Capital Income	0.67	0.59
Corr (cap inc., lab inc.)	0.80	0.50
Persistence of inc.	0.81	0.74

Table 2.9: Changes in Output-From benchmark to $\phi = 1$

	$\rho = 0.1$
Output Change (%)- Baseline	48
Output Change (%)- No Labor Heterogeneity	48
Output Change (%)- No Self-Employed	54

Table 2.10: Changes in Output-From Benchmark to $\phi = 1$

	ρ		
	-0.8	0.1	0.8
Output Change (%)- Baseline	55	48	37
Output Change (%)- No Labor Heterogeneity	47	48	41
Output Change (%)- No Self-Employed	63	54	44

Table 2.11: Skill Correlation ρ and the Selection of Entrepreneurs

	ρ		
	-0.8	0.1	0.8
Output Change (%)- Baseline	55	48	37
Output Change (%)- No Labor Heterogeneity	47	48	41
Output Change (%)- No Self-Employed	63	54	44

Table 2.12: Statistics on returns for different occupations and TFP gains

	$\rho_{w,m} = 0.1$
TFP gains (%)	15
Employers	
Std. Dev. MPK_e	0.10
Std. Dev. c_e	0.19
$corr(c_e, z_m)$	-0.55
Self-Employed	
Std. Dev. MPK_{se}	0.06
Std. Dev. c_{se}	0.07
$corr(c_{se}, z_m)$	-0.30

For $\phi = 1$ the standard deviation is 0 for all variables

c_e is the return to managerial input for employers

c_{se} is the return to the composite input for self-employed

MPK_e, MPK_{se} are the marginal product of capital for employers and self-employed respectively

Table 2.13: Implications of ρ for earnings and consumption

	Data	Model
Ratio Median Earnings Worker to Entrepreneur	1.0	1.0
$corr(earnings_t, earnings_{t-1})$	N.A.	0.81
$corr(earnings_t, consumption_t)$	0.71	0.79

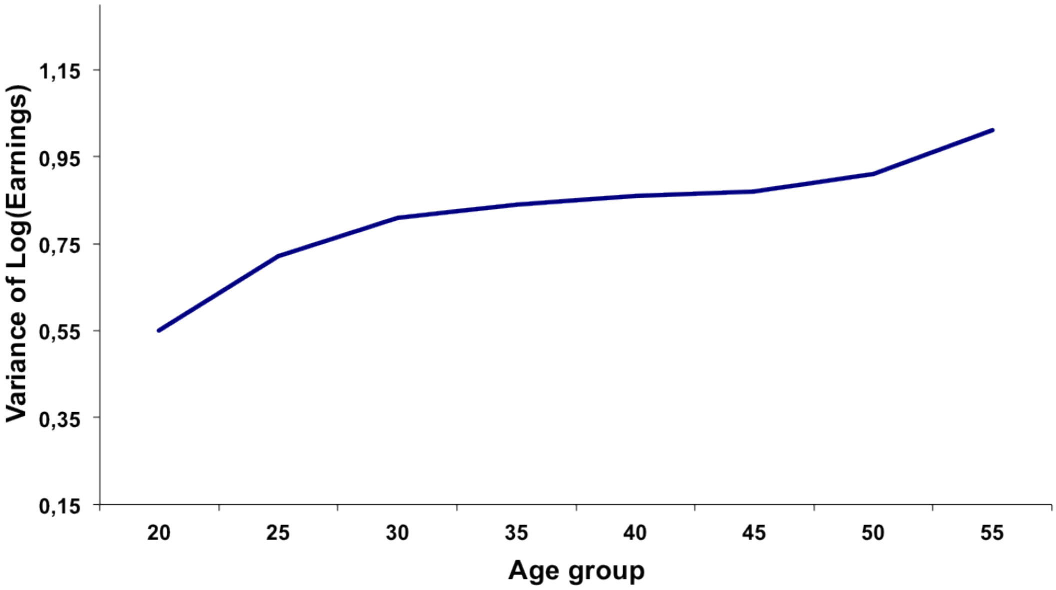
Table 2.14: Gini Inex-Labor and Capital Income

	Gini Index	
	$\phi = 0.23$	$\phi = 1$
Labor Income	0.52	0.56
Capital Income	0.67	0.59

Table 2.15: Consumption Inequality and Financial Frictions

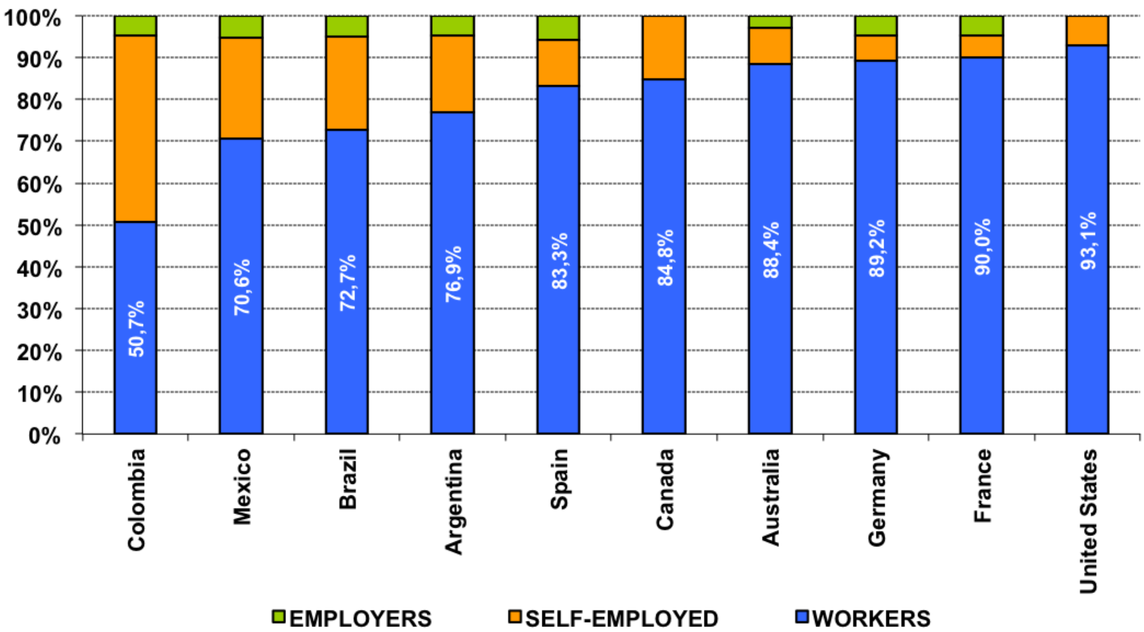
	$\phi = 0.23$	$\phi = 1$
Gini	0.50	0.48
p90/p10	11.5	10.33
p90/p50	3.29	3.44
p10/p50	0.29	0.33

Figure 2.1: Variance of Log Earnings in Brazil



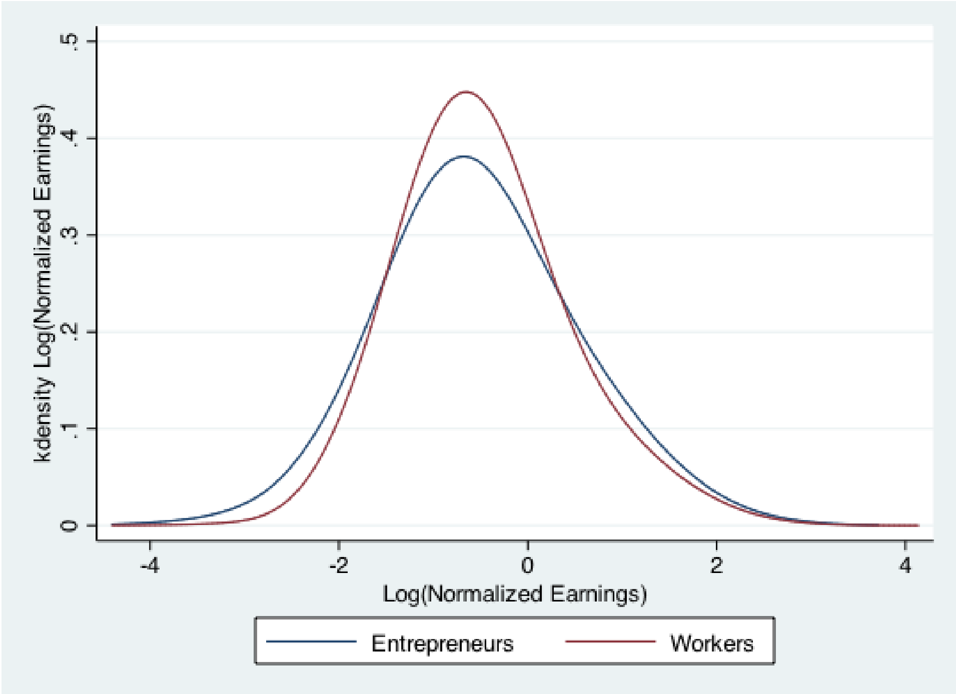
Source: Author's Elaboration based on PME 2003-2010

Figure 2.2: Occupational Structure across Countries



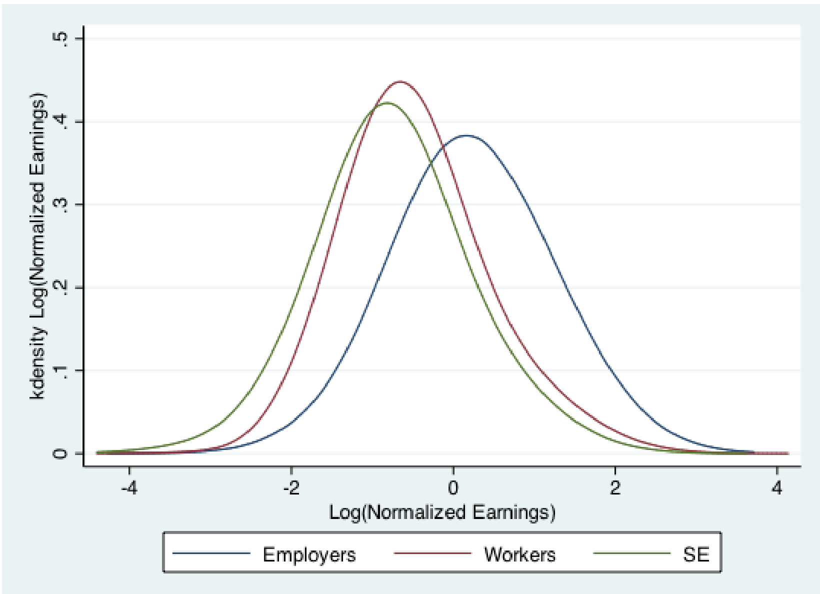
Source: Author's Elaboration based on ILO 2008

Figure 2.3: Distribution of Earnings in Brazil by Occupation-I



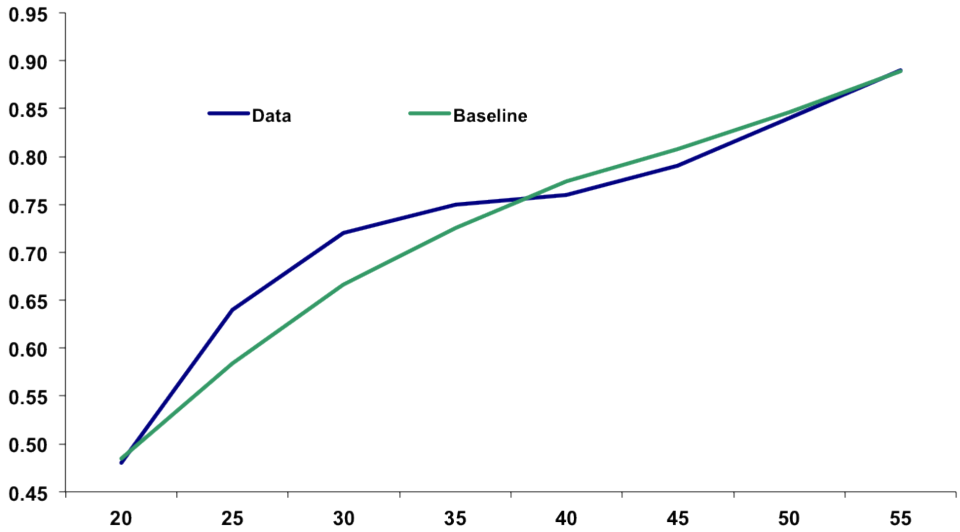
Source: Author's Elaboration based on PME 2003-2010

Figure 2.4: Distribution of Earnings in Brazil by Occupation-II



Source: Author's Elaboration based on PME 2003-2010

Figure 2.5: Variance of Log(Earnings)-Model vs Data



Source: Author's Elaboration based on PME 2003-2010

Figure 2.6: Distribution of Earnings-Data vs Model I

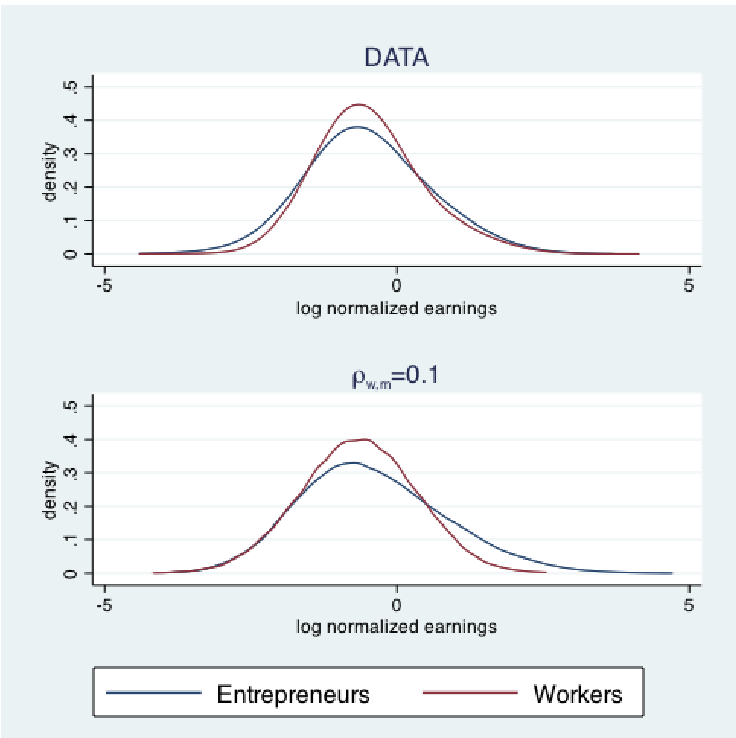


Figure 2.7: Distribution of Earnings-Data vs Model II

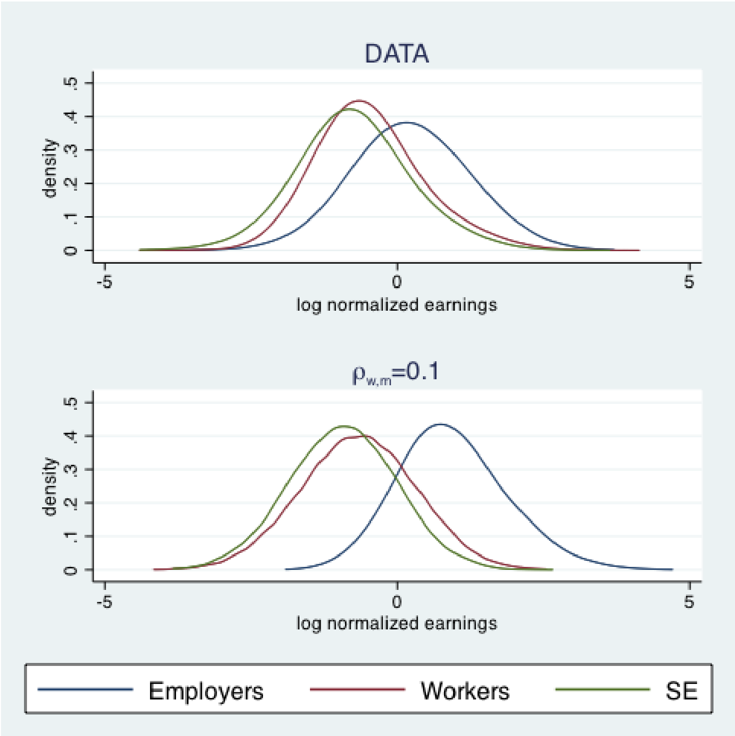


Figure 2.8: MPK and Market Return on Capital

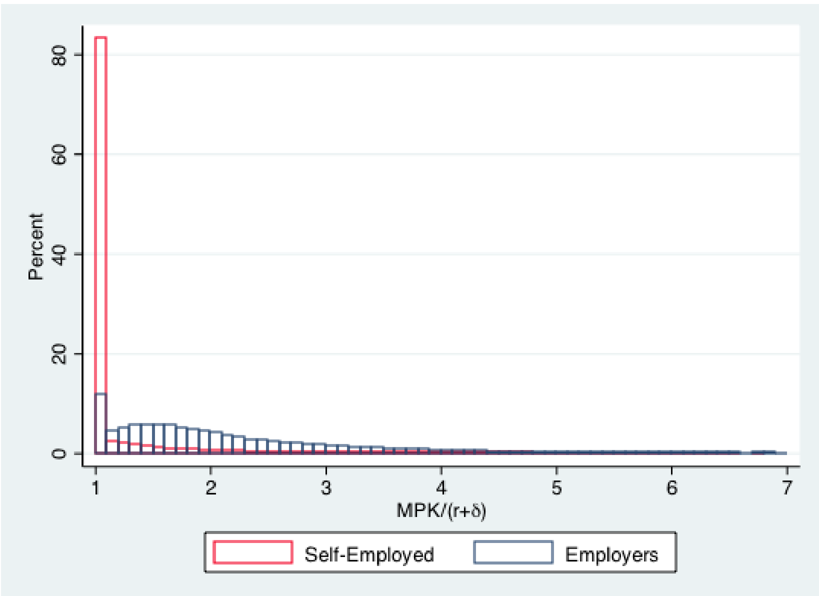


Figure 2.9: Correlation between $\frac{z_m}{z_w}$ and z_w for different $\rho_{w,m}$

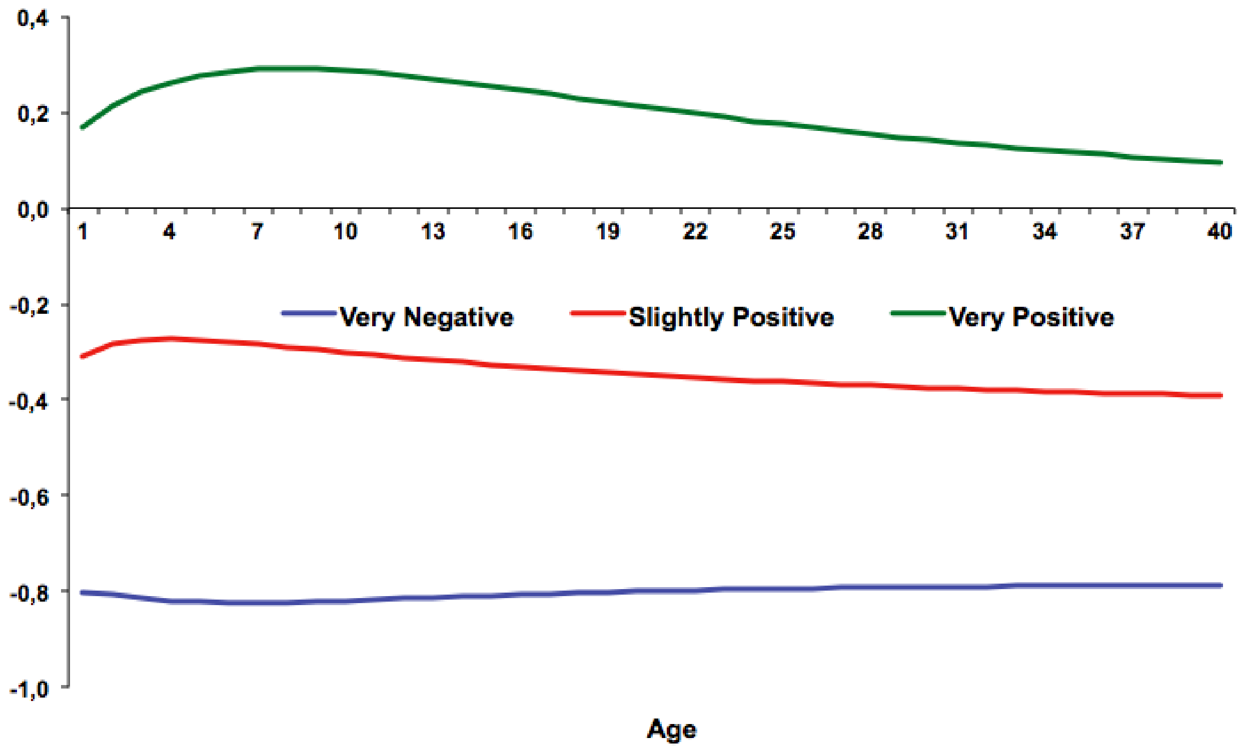


Figure 2.10: Occupational Map-$\phi = 1$

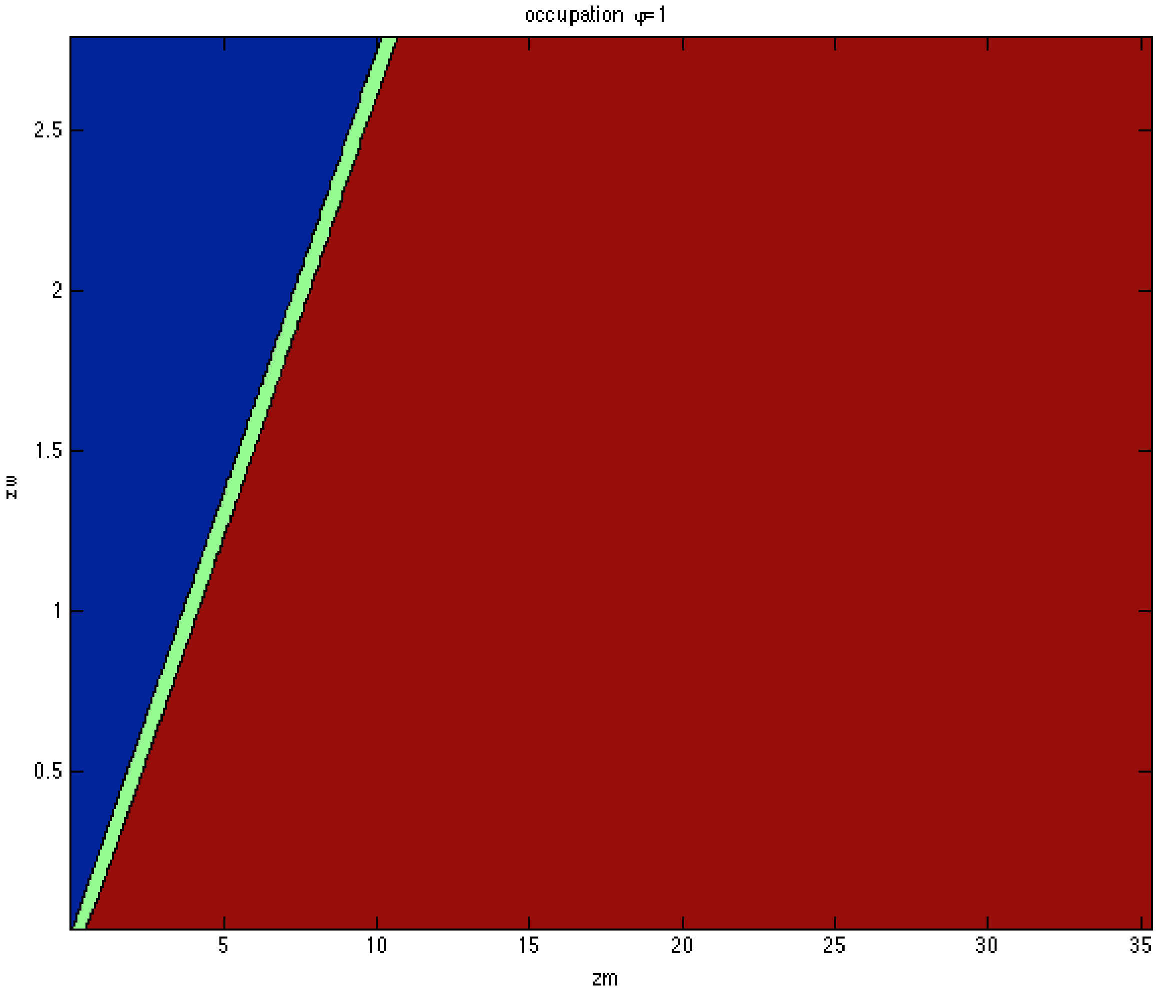


Figure 2.11: Occupational Maps Benchmark Economy

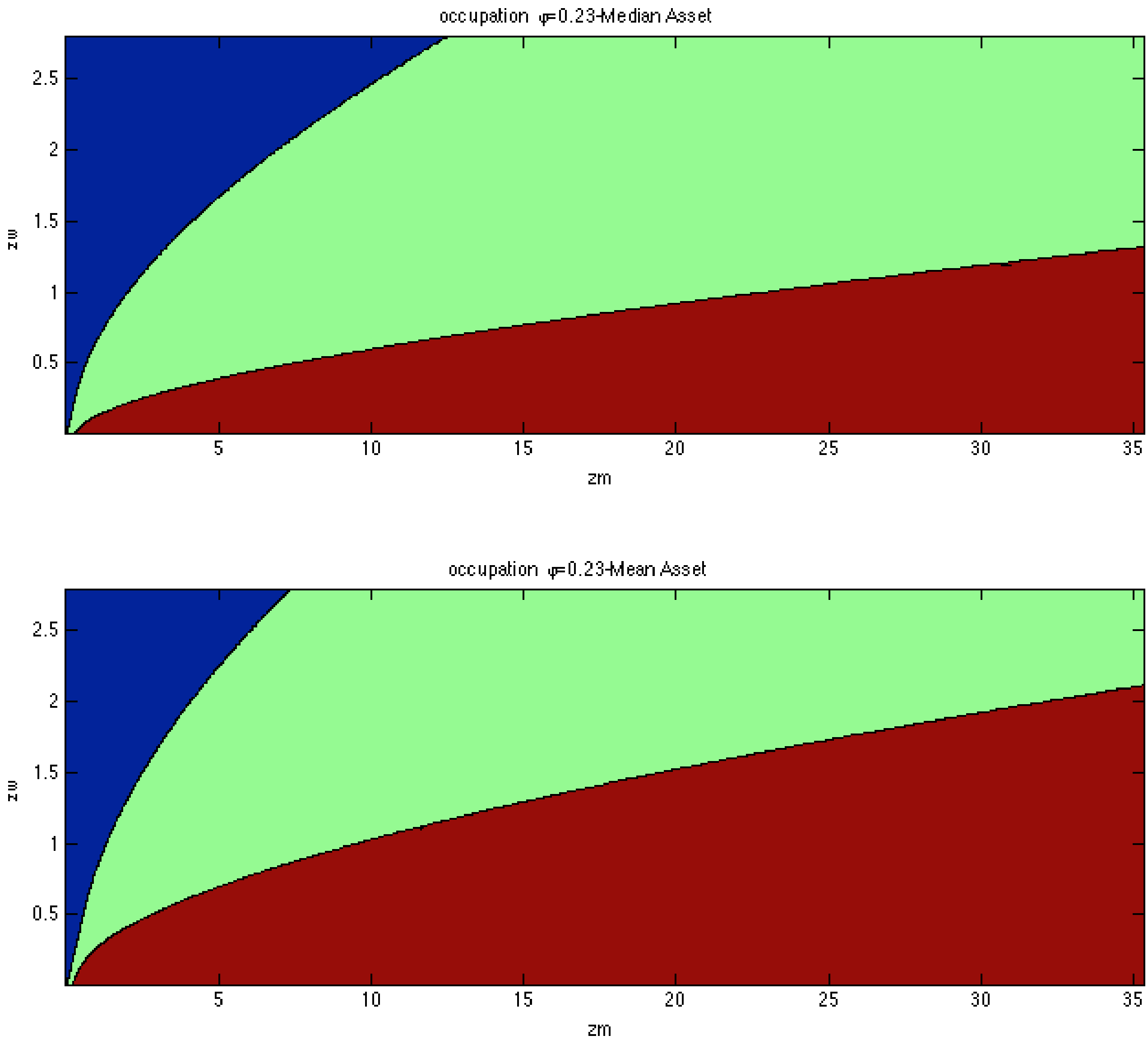


Figure 2.12: Distribution of Earnings-Data vs Model I

Figure 2.13: Distribution of Earnings-Data vs Model II

Figure 2.14: Welfare Gains from Financial Reform

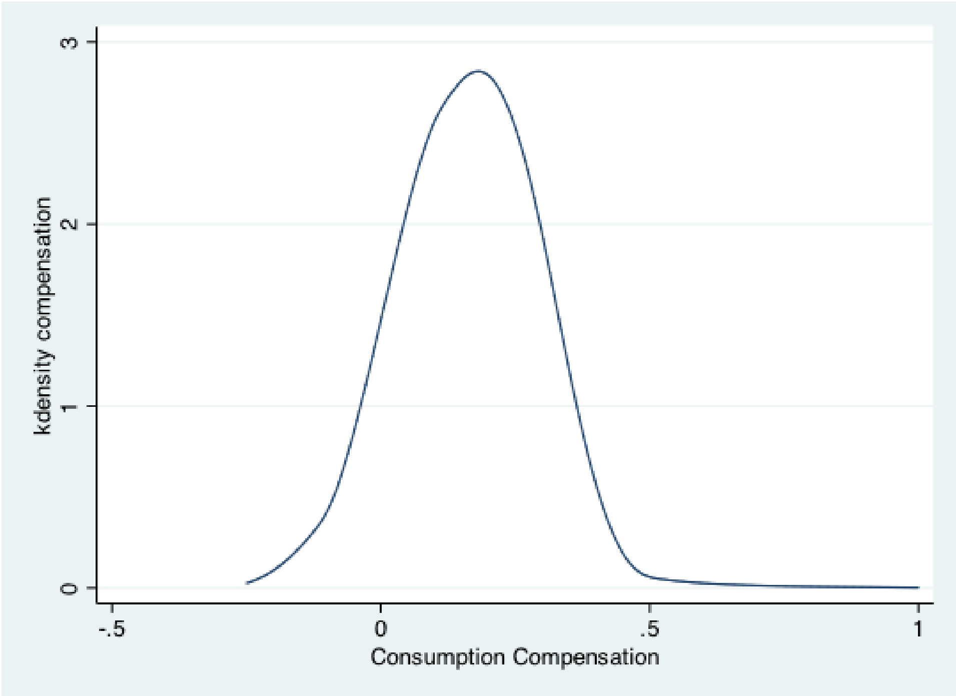


Figure 2.15: Age Distribution of Winners and Losers from the Reform

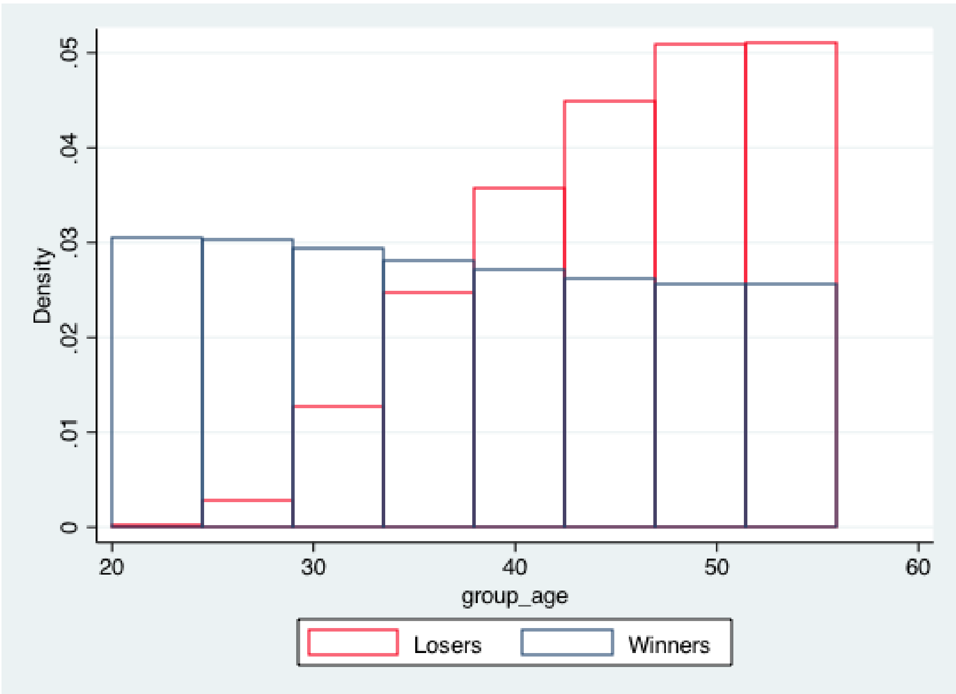


Figure 2.16: Wealth Distribution and the Reform

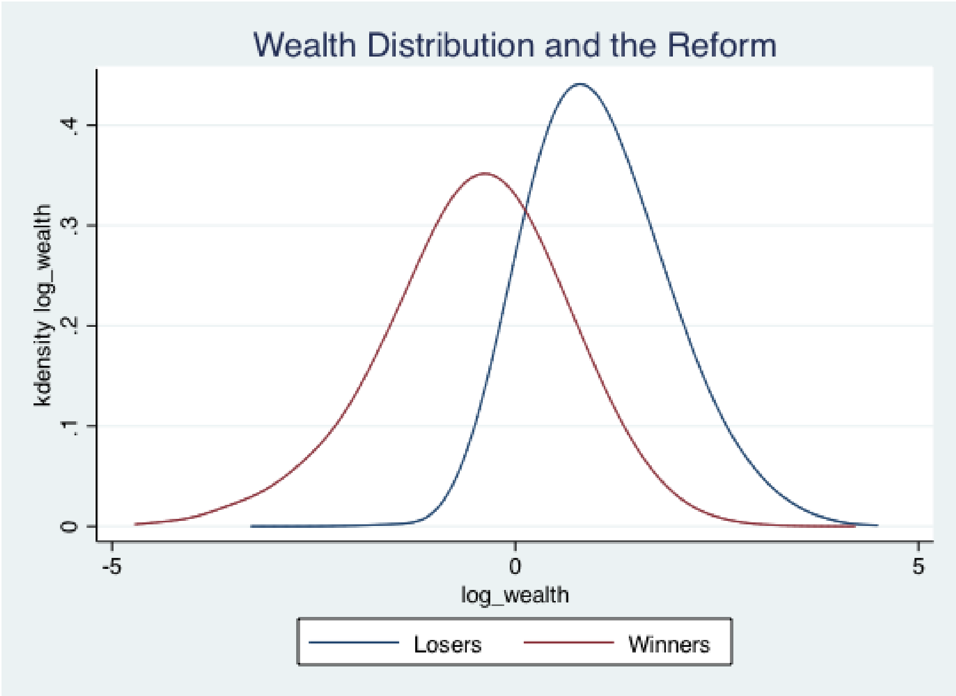


Figure 2.17: Distribution of Managerial Ability and the Reform

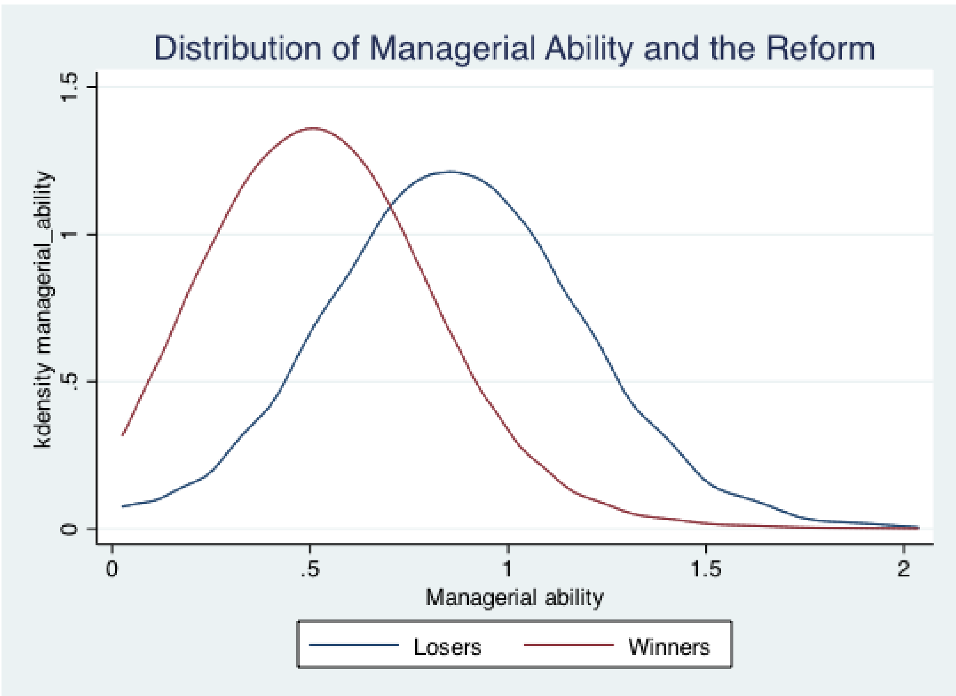
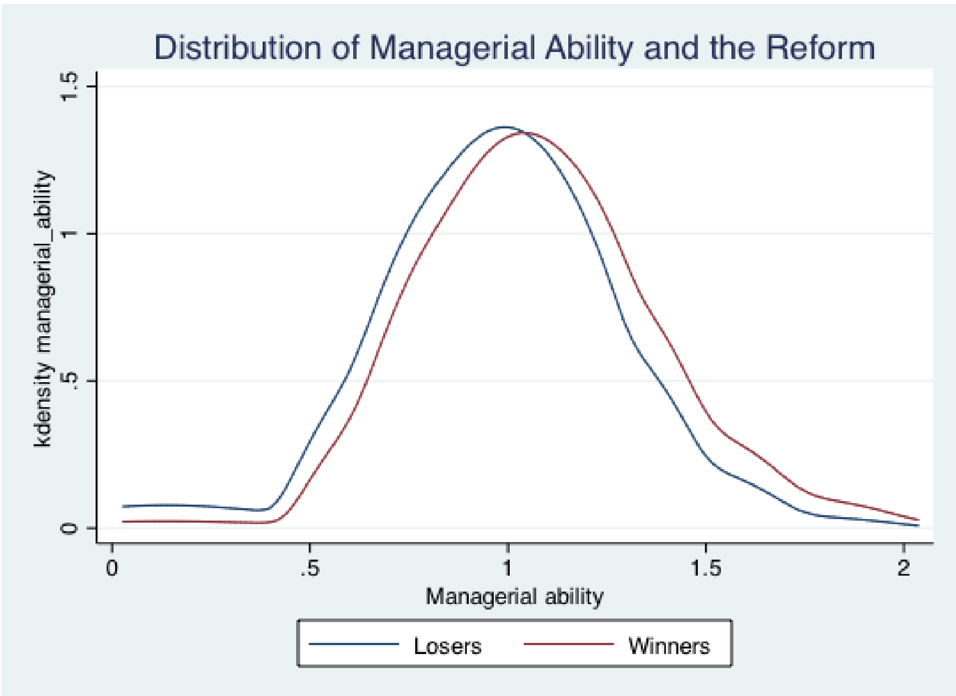


Figure 2.18: Distribution of Managerial Ability and the Reform among Employers



Appendix A

Appendix to Chapter 1

Appendix A.1: Labor for the smaller firm operating

The amount of labor demanded by the smaller firm is:

$$\begin{aligned}
\ell(\phi^*_{iii}) &= \frac{q(\phi^*_{iii})}{\phi^*_{iii}} + \kappa^d_i \\
q(\phi^*_{iii}) &= \frac{r(\phi^*_{iii})}{p(\phi^*_{iii})} \\
\text{from equation (\textbf{??})} \rightarrow r(\phi^*_{iii}) &= \sigma w_i \kappa^d_i \\
\text{and from equation (1.5)} \rightarrow p(\phi^*_{iii}) &= \frac{\sigma}{\sigma - 1} \frac{w_i}{\phi^*_{iii}} \\
\text{then} \rightarrow q(\phi^*_{iii}) &= (\sigma - 1)\kappa^d_i \phi^*_{iii} \\
\ell(\phi^*_{iii}) &= \sigma \kappa^d_i
\end{aligned}$$

Appendix A.2: Aggregation

In this section I will show how to get the weighted average productivity of firms producing and selling in each country, as well as the aggregate price and production.

From equation 1.22 we obtain the total mass of firms producing and the total mass of firms selling in country i. Let us define the average productivity of firms performing each activity:

$$
\begin{aligned}
\tilde{\phi}_{iii} &= \left[\int_{\phi^*_{iii}}^{\infty} \phi^{\sigma-1}\mu_i d\phi\right]^{\frac{1}{1-\sigma}} \\
\tilde{\phi}_{kii} &= \left[\int_{\phi^*_{kii}}^{\infty} \phi^{\sigma-1}\mu_i d\phi\right]^{\frac{1}{1-\sigma}} \\
\tilde{\phi}_{kki} &= \left[\int_{\phi^*_{kki}}^{\infty} \phi^{\sigma-1}\mu_i d\phi\right]^{\frac{1}{1-\sigma}} \\
\tilde{\phi}_{jki} &= \left[\int_{\phi^*_{jki}}^{\infty} \phi^{\sigma-1}\mu_i d\phi\right]^{\frac{1}{1-\sigma}}
\end{aligned}
$$

Using the expressions from above we can define the weighted average productivity as:

$$
\begin{aligned}
\tilde{\phi}^p_i &= \left\{\frac{1}{M^p_i}\left[M_{iii}\tilde{\phi}^{\sigma-1}_{iii} + \sum_{k\neq i} M_{kii}\frac{E^1_k}{E^1_i}\left(\frac{P_k}{\tau_{ki}P_i}\right)^{\sigma-1}\tilde{\phi}^{\sigma-1}_{kii} + \sum_{k\neq i} M_{iik}\left(\frac{1}{\gamma_{ik}}\right)^{\sigma-1}\tilde{\phi}^{\sigma-1}_{iik}\right.\right. \\
&+ \left.\left.\sum_{k\neq i}\sum_{i\neq j} M_{kij}\frac{E^1_k}{E^1_i}\left(\frac{P_k}{\tau_{ki}\gamma_{ij}}\right)^{\sigma-1}\tilde{\phi}^{\sigma-1}_{kij}\right]\right\}^{\frac{1}{\sigma-1}}, \qquad \text{(A.1)} \\
\tilde{\phi}^s_i &= \left\{\frac{1}{M^s_i}\left[M_{iii}\tilde{\phi}^{\sigma-1}_{iii} + \sum_{k\neq i} M_{ikk}\left(\frac{w_k\tau_{ik}}{w_i}\right)^{1-\sigma}\tilde{\phi}^{\sigma-1}_{ikk} + \sum_{k\neq i} M_{iik}\gamma^{1-\sigma}_{ik}\tilde{\phi}^{\sigma-1}_{iik}\right.\right. \\
&+ \left.\left.\sum_{k\neq i}\sum_{i\neq j} M_{ijk}\left(\frac{\tau_{ij}\gamma_{jk}w_k}{w_i}\right)^{1-\sigma}\tilde{\phi}^{\sigma-1}_{ijk}\right]\right\}^{\frac{1}{\sigma-1}}. \qquad \text{(A.2)}
\end{aligned}
$$

Let us write now the equation for aggregate price in country i (equation 1.20)

$$
\begin{aligned}
P_i &= \left[\int_{\phi^*_{iii}} (p_{iii}(\phi))^{1-\sigma} M_i\mu_i(\phi)d\phi + \sum_{k\neq i}\int_{\phi^*_{ikk}} (p_{ikk}(\phi))^{1-\sigma} M_k\mu_k(\phi)d\phi\right. \qquad \text{(A.3)} \\
&+ \left.\sum_{k\neq i}\int_{\phi^*_{iik}} (p_{iik}(\phi))^{1-\sigma} M_k\mu_k(\phi)d\phi + \sum_{k\neq j}\sum_{k\neq i}\int_{\phi^*_{ikj}} (p_{ikj}(\phi))^{1-\sigma} M_j\mu_j(\phi)d\phi\right]^{\frac{1}{1-\sigma}}.
\end{aligned}
$$

now, replace $p_{ikj}(\phi)$ $\forall i, j, k$ using equation 1.5 in the previous expression to obtain:

$$\begin{aligned}
P_i &= \left[\int_{\phi^*_{iii}} \left(\frac{w_i}{\rho\phi}\right)^{1-\sigma} M_i\mu_i(\phi)d\phi + \sum_{k\neq i}\int_{\phi^*_{ikk}} \left(\frac{w_k\tau_{ik}}{\rho\phi}\right)^{1-\sigma} M_k\mu_k(\phi)d\phi \right.\\
&+ \left.\sum_{k\neq i}\int_{\phi^*_{iik}} \left(\frac{w_i\gamma_{ik}}{\rho\phi}\right)^{1-\sigma} M_k\mu_k(\phi)d\phi + \sum_{k\neq j}\sum_{k\neq i}\int_{\phi^*_{ikj}} \left(\frac{w_k\gamma_{kj}\tau_{ik}}{\rho\phi}\right)^{1-\sigma} M_j\mu_j(\phi)d\phi\right]^{\frac{1}{1-\sigma}}\\
P_i &= \left[\left(\frac{w_i}{\rho}\right)^{1-\sigma} M_i\int_{\phi^*_{iii}} (\phi)^{\sigma-1}\mu_i(\phi)d\phi + \sum_{k\neq i} M_{ikk}\left(\frac{w_k\tau_{ik}}{\rho}\right)^{1-\sigma}\int_{\phi^*_{ikk}} (\phi)^{\sigma-1}\mu_k(\phi)d\phi\right.\\
&+ \left.\sum_{k\neq i} M_{iik}\left(\frac{w_i\gamma_{ik}}{\rho}\right)^{1-\sigma}\int_{\phi^*_{iik}} (\phi)^{\sigma-1}\mu_k(\phi)d\phi + \sum_{k\neq j}\sum_{k\neq i} M_{ikj}\left(\frac{w_k\gamma_{kj}\tau_{ik}}{\rho}\right)^{1-\sigma}\int_{\phi^*_{ikj}} (\phi)^{\sigma-1}\mu_j(\phi)d\phi\right]^{\frac{1}{1-\sigma}}
\end{aligned}$$

We can replace the integral terms by each of the average productivities, and we get:

$$\begin{aligned}
P_i &= \frac{w_i}{\rho}\left[M_{iii}\tilde{\phi}_{iii}^{1-\sigma} + \sum_{k\neq i} M_{ikk}\left(\frac{w_k\tau_{ik}}{w_i}\right)^{1-\sigma}\tilde{\phi}_{ikk}^{1-\sigma}\right.\\
&+ \left.\sum_{k\neq i} M_{iik}\gamma_{ik}^{1-\sigma}\tilde{\phi}_{iik}^{1-\sigma} + \sum_{k\neq j}\sum_{k\neq i} M_{ikj}\left(\frac{w_k\gamma_{kj}\tau_{ik}}{w_i}\right)^{1-\sigma}\tilde{\phi}_{ikj}^{1-\sigma}\right]^{\frac{1}{1-\sigma}}
\end{aligned}$$

Note that the term inside brackets is $\frac{(M_i^s)^{\frac{1}{\sigma-1}}}{\tilde{\phi}_i^s}$, and that $p(\tilde{\phi}_i^s) = \frac{w_i}{\rho\tilde{\phi}_i^s}$. Then

$$P = (M_i^s)^{\frac{1}{1-\sigma}} p(\tilde{\phi}_i^s)$$

In a similar way we can derive the equation for aggregate GDP.

$$\begin{aligned}
GDP_i &= \int_{\phi^*_{iii}} r_{iii}(\phi)M_i\mu_i d\phi + \sum_{k\neq i}\int_{\phi^*_{kii}} r_{kii}(\phi)M_i\mu_i d\phi\\
&+ \sum_{k\neq i}\int_{\phi^*_{iik}} r_{iik}(\phi)M_k\mu_k d\phi + \sum_{k\neq j}\sum_{k\neq i}\int_{\phi^*_{kij}} r_{kij}(\phi)M_j\mu_j d\phi
\end{aligned}$$

Replacing $r(\phi)$ by the expressions found in equation 1.6 we get:

$$\begin{aligned}
GDP_i &= \int_{\phi^*_{iii}} E^1_i P^{\sigma-1}_i \left(\frac{\rho\phi}{w_i}\right)^{\sigma-1} M_i \mu_i d\phi + \sum_{k\neq i} \int_{\phi^*_{kii}} E^1_k P^{\sigma-1}_k \left(\frac{\rho\phi}{w_i \tau_{ki}}\right)^{\sigma-1} M_i \mu_i d\phi \\
&+ \sum_{k\neq i} \int_{\phi^*_{iik}} E^1_i P^{\sigma-1}_i \left(\frac{\rho\phi}{w_i \gamma_{ik}}\right)^{\sigma-1} M_k \mu_k d\phi \\
&+ \sum_{k\neq j} \sum_{k\neq i} \int_{\phi^*_{kij}} E^1_k P^{\sigma-1}_k \left(\frac{\rho\phi}{w_i \gamma_{ij} \tau_{ki}}\right)^{\sigma-1} M_j \mu_j d\phi \\
GDP_i &= E^1_i P^{\sigma-1}_i \left(\frac{\rho}{w_i}\right)^{\sigma-1} M_i \int_{\phi^*_{iii}} \phi^{\sigma-1} \mu_i d\phi + \sum_{k\neq i} E^1_k P^{\sigma-1}_k \left(\frac{\rho}{w_i \tau_{ki}}\right)^{\sigma-1} M_i \int_{\phi^*_{kii}} \phi^{\sigma-1} \mu_i d\phi \\
&+ \sum_{k\neq i} E^1_i P^{\sigma-1}_i \left(\frac{\rho}{w_i \gamma_{ik}}\right)^{\sigma-1} M_k \int_{\phi^*_{iik}} \phi^{\sigma-1} \mu_k d\phi \\
&+ \sum_{k\neq j} \sum_{k\neq i} E^1_k P^{\sigma-1}_k \left(\frac{\rho}{w_i \gamma_{ij} \tau_{ki}}\right)^{\sigma-1} M_j \int_{\phi^*_{kij}} \phi^{\sigma-1} \mu_j d\phi
\end{aligned}$$

We can replace again the integral terms by the average productivities for each occupation, and operating we get:

$$\begin{aligned}
GDP_i &= E^1_i P^{\sigma-1}_i \left(\frac{\rho}{w_i}\right)^{\sigma-1} \left[M_i \tilde{\phi}^{1-\sigma}_{iii} + \sum_{k\neq i} \frac{E^1_k}{E^1_i} \left(\frac{P_k}{P_i w_i \tau_{ki}}\right)^{\sigma-1} M_{kii} \tilde{\phi}^{1-\sigma}_{kii} \right. \\
&+ \left. \sum_{k\neq i} \left(\frac{1}{\gamma_{ik}}\right)^{\sigma-1} M_{iik} \tilde{\phi}^{1-\sigma}_{iik} \sum_{k\neq j} \sum_{k\neq i} \frac{E^1_k}{E^1_i} \left(\frac{P_k}{P_i \gamma_{ij} \tau_{ki}}\right)^{\sigma-1} M_j \tilde{\phi}^{1-\sigma}_{kij} \right]
\end{aligned}$$

Note that the term in brackets is equal to $M^p_i * \left(\tilde{\phi}^p_i\right)^{\sigma-1}$, then

$$GDP_i = M^p_i E^1_i P^{\sigma-1}_i \left(\frac{\rho}{w_i}\right)^{\sigma-1} \left(\tilde{\phi}^p_i\right)^{\sigma-1}$$

and as $r_{iii}(\tilde{\phi}^p_i) = E^1_i P^{\sigma-1}_i \left(\frac{\rho\tilde{\phi}^p_i}{w_i}\right)^{\sigma-1}$ then

$$GDP_i = M^p_i r_{iii}(\tilde{\phi}^p_i)$$

Appendix A.3: Algorithm to calculate the cut-offs

In order to calculate the productivity cut-offs for each activity, you need to guess N domestic cut-offs (ϕ^*_{iii}) and with these you can compute for each country i, the N exporting cut-offs (ϕ^*_{kii}), the N MP cut-offs (ϕ^*_{kki}), and the NxN BMP cut-offs (ϕ^*_{jki}) using equation 1.7, 1.10, and 1.13.

Once we have all the cut-offs computed we need to follow the next steps for each country. Take country i:

1. Check that the MP cut-offs (ϕ^*_{kki}) are well computed.
   (a) If all the BMP cut-offs for firms from country i producing in country k are bigger than the MP cut-offs for the same firms ($\phi^*_{kki} < \phi^*_{jki}\ \forall\ j$), then the MP cut-offs are well computed and you have to go to step 2.
   (b) If at least one BMP cut-off is smaller than the MP cut-off, then:
      i. Take the smallest BMP cut-off for firms from country i producing in country k and re-calculated the MP cut-off using equation 1.14.
      ii. If the new MP cut-off is smaller than the second lowest BMP cut-off, then this is the MP cut-off, otherwise, take the two smallest BMP cut-offs and re-calculated the MP cut-off using equation 1.14.
      iii. Check that the new MP-cut-off is smaller than the rest BMP cut-offs or repeat the previous step incorporating the next BMP cut-off until the there are no more BMP cut-offs smaller than the MP cut-off.
      iv. Go to step 2
2. Check that the domestic cut-offs are well computed. Order all the exporting cut-offs and MP cut-offs for country i, from the smallest to the biggest. To simplify the explanation, assume that the smallest cut-off is the exporting cut-off to country k (ϕ^*_{kki}) (it can be that an MP cut-off is the smallest, and the procedure will be the same).
   (a) If the exporting cut-off from country i to country k is above the domestic cut-off (ϕ^*_{iii}), then the domestic cut-off is well computed and we are done.
   (b) If the exporting cut-off to country k is below the domestic cut-off, then:
      i. Take the exporting cut-off from country i to country k and the domestic cut-off from country i and compute the new domestic cut-off using equation firms from country i producing in country k and re-calculated the MP cut-off using equation 1.12.
      ii. If the new domestic cut-off is smaller than the second smallest cut-off (from the exporting cut-off and MP cut-off set), then this is the domestic cut-off, otherwise, take the two smallest cut-offs and re-calculated the domestic cut-off using equation 1.12.
      iii. Check that the new domestic cut-off is smaller than the rest of cut-offs (exporting or MP) or repeat the previous step incorporating the next cut-off until there are no more cut-offs (exporting or MP) smaller than the domestic cut-off.

Appendix B

Appendix to Chapter 2

Appendix B.1: Data

Pesquisa Mensal de Emprego From this survey we have data for the years 2003 until 2010. The PME is a monthly household survey covering the metropolitan areas of six Brazilian regions: Rio de Janeiro, São Paulo, Porto Alegre, Belo Horizonte, Recife and Salvador. Each individual is followed for three months, left out of the sample the next eight months and interviewed again the following 4 months. We take the first and fifth interview of each individual for the years 2003 until 2010. In this way we keep two observation of each individual, which corresponds to the same month of consecutive years. We keep only household where the head is male and he is older than twenty and younger than sixty years old. The earnings of the household are the sum of the earnings of all members. In order to make the earnings comparable we deflect them with the corresponding month Consumer Price Index (CPI) and we divide them by the number of adults equivalents in the house. In addition, we only keep individuals who are employed in both periods of the survey. In the final data set we have 131,056 households with data for earnings. Individual households age is defined as the age of the household head. We use 5 years bin, centered at the age of interested, in order to compute statistics by age. To do the transition matrix of employment we consider the individual data. The variable of earnings that we consider is a constructed variable, which includes the earnings effectively perceived by the individual in the month from all the works done.

Pesquisa de Ornamentos Familiares The POF is a Consumption-Income survey done every five or six years. We use data from the last wave, 2008-2009. We consider households where the main earner is a male, older than twenty and younger than sixty years old. We end up with 44,930 observations. Our income variable includes: income from work, Transfers, Income from rents, other and Asset Variation. Our measure of consumption includes: food, housing, clothing, transport, health and personal care, education, recreation and culture, smoking, personal services and other current expenses. We normalize household income and consumption by dividing them by the number of adults equivalents in the house.

Appendix B.2: Proofs

Proof of Proposition 1. Capital rental k by an entrepreneur with wealth a and skills (z_m, z_w) is enforceable if and only if

$$\max_{m,n,n_d,t_e} \{m^\gamma k^\nu n^\theta - wn^d - r(k-a) + a - \delta k - c_f \mathrm{I}_{n_d>0}\} \geq \\ \max_{m,n,n_d,t_m} \{m^\gamma k^\nu n^\theta - wn^d + (1-\delta)k - c_f \mathrm{I}_{n_d>0}\}$$

which is equivalent to

$$(1+r)a \geq \phi \left[\frac{1-\phi+r+\delta+\delta\phi}{\phi} k - \phi \max_{m,n,n_d,t_e} \{m^\gamma k^\nu n^\theta - wn^d - c_f \mathrm{I}_{n_d>0}\} \right]$$

Following arguments in Buera, Kaboski, and Shin (2011), the set of enforceable levels of capital rentals is characterized by a simple set of rental limits. Two cases are relevant. If the max in the RHS is attained with $n_d = 0$, the set of enforceable levels of capital is $[0, \overline{k}(a, z_m, z_w; \phi)]$ where $\overline{k}(a, z_m, z_w; \phi)$, where $\overline{k}(a, z_m, z_w; \phi)$ is given by unique root of the equation

$$(1+r)a = \phi \left[\frac{1-\phi+r+\delta+\delta\phi}{\phi} k - \phi \max_{m,n,n_d,t_e} \{m^\gamma k^\nu n^\theta - wn^d - c_f \mathrm{I}_{n_d>0}\} \right]$$

If the max in the RHS is attained with $n_d > 0$, then there are two positive roots of the above equation and the set of enforceable levels of capital rental is $[\underline{k}(a, z_m, z_w; \phi), \overline{k}(a, z_m, z_w; \phi)]$, where $\underline{k}(a, z_m, z_w; \phi)$ represents the smallest root. Nonetheless, the optimal production plan of the entrepreneur coincides with the solution to the individual problem subject to the simpler limit $k \leq \overline{k}(a, z_m, z_w; \phi)$. It can also be shown that $\overline{k}(a, z_m, z_w; \phi)$ is strictly increasing in a, z_m, ϕ and weakly (strictly) increasing in z_w (if $n_d = 0$).

Proof of Proposition 2. The optimal production plan of self-employed individuals solve

$$\pi^{se} = (z_m t_m)^\gamma k^\nu (z_w(1-t_m))^\theta - (r+\delta)k + (1+r)a + \mu_k(\overline{k} - k)$$

where $\gamma + \nu + \theta = 1$. The FOC imply:

$$\{t_m\} \qquad z_m^\gamma k^\nu z_w^\theta [\gamma t_m^{\gamma-1}(1-t_m)^\theta - t_m^\gamma \theta (1-t_m)^{\theta-1}] = 0 \Rightarrow t_m^* = \frac{\gamma}{\gamma+\theta}$$

$$\{k\} \qquad (z_m t_m)^\gamma \nu k^{\nu-1} (z_w(1-t_m))^\theta - r - \delta - \mu_k = 0 \Rightarrow k = \left[\frac{(z_m t_m)^\gamma \nu (z_w(1-t_m))^\theta}{r+\delta+\mu_k} \right]^{\frac{1}{1-\nu}}.$$

Note that the first FOC equates the marginal product of entrepreneurial time at managing and worker. Combining the FOC we obtain that the marginal product of entrepreneurial time satisfies:

$$\begin{aligned} MPT_{se} &= \gamma z_m^{\gamma} \left(t_m^*\right)^{\gamma-1} k^{*\nu} \left(z_w(1-t_m^*)\right)^{\theta} \\ &= r_{mw} \left(z_m^{\gamma} z_w^{\theta}\right)^{\frac{1}{\gamma+\theta}}, \\ \text{where} \quad & r_{mw} = \gamma\, \nu^{\frac{\nu}{1-\nu}} \left(\frac{\gamma\,\theta}{(\gamma+\theta)^2}\right)^{\frac{\theta}{1-\nu}} \left(\frac{1}{r+\delta+\mu}\right)^{\frac{\nu}{1-\nu}}. \end{aligned}$$

Income of self-employed individuals can then be written as

$$\begin{aligned} y_{se} &= MPt_m\, t_m + MPt_w\, t_w + MPK\, k + ra - k(r+\delta), \\ y_{se} &= MPT_{se} \times 1 + (r+\mu+\delta)k + ra - k(r+\delta), \\ y_{se} &= r_{mw} \left(z_m^{\gamma} z_w^{\theta}\right)^{\frac{1}{\gamma+\theta}} + \mu k + ra. \end{aligned}$$

Proof of Proposition 3. An individual with ability (z_m, z_w) prefers to be self-employed rather than work for a wage if and only if

$$z_w w + ra < \left(z_m^{\gamma} z_w^{\theta}\right)^{\frac{1}{\theta+\gamma}} r_{mw} + \mu k + ra,$$

which holds when the skill ratio satisfies

$$\frac{z_w}{z_m} < \left[\frac{r_{mw} + \mu k / \left(z_m^{\gamma} z_w^{\theta}\right)^{\frac{1}{\theta+\gamma}}}{w}\right]^{\frac{\theta+\gamma}{\gamma}}.$$

If capital markets are perfect ($\phi = 1$), the Lagrange multiplier on the borrowing constraint is equal to zero ($\mu = 0$) and the individual prefers to be self-employed rather than work for a wage if and only if

$$\frac{z_w}{z_m} < \left(\frac{r_{mw}}{w}\right)^{\frac{\theta+\gamma}{\gamma}} \equiv R_1.$$

Proof of Proposition 4. The optimal production plan of employers solves

$$\begin{aligned} \pi(z_m, z_w, a) &= Max_{t_m, t_w, n_d, k} (z_m t_m)^{\gamma} k^{\nu} (n_d + z_w t_w)^{\theta} - w n_d - (r+\delta)k + (1+r)a \\ & k \le \overline{k} \\ & t_m + t_w = 1, \\ & t_w \ge 0. \end{aligned}$$

The non-negativity constraint on t_w ensures that managerial time cannot be bigger than 1. Associate the multiplier μ_k to the borrowing constraint, μ_t to the time constraint, and μ_{tw} to the non-negative constraint on the working time. The FOC of the problem imply

$$\begin{aligned}
MPK &= (z_m t_m)^{\gamma} \nu k^{\nu-1} (n_d + z_w t_w)^{\theta} = r + \delta + \mu_k, \\
MPn_d &= (z_m t_m)^{\gamma} k^{\nu} \theta (n_d + z_w t_w)^{\theta-1} = w, \\
MPt_m &= z_m \gamma (z_m t_m)^{\gamma-1} k^{\nu} (n_d + z_w t_w)^{\theta} = \mu_t, \\
MPt_w &= (z_m t_m)^{\gamma} \theta k^{\nu} (n_d + z_w t_w))^{\theta-1} z_w = \mu_t - \mu_{tw},
\end{aligned}$$

where we have assumed that parameters are such that it is optimal to hire outside labor ($n_d > 0$). Combining the FOC we obtain:

$$w z_w = MPt_w \leq MPt_m, \text{with equality only if } t_w > 0.$$

We divide the analysis in two steps.

Step 1: We first show that if the borrowing constraint does not bind ($\mu_k = 0$), then the entrepreneur allocate all his time to managerial tasks ($t_w = 0, t_m = 1$). Assume that $\mu_k = 0$ and let $L \equiv n_d + z_w(1 - t_m)$. Furthermore, to find a contradiction assume that $t_w > 0$. Then, $\mu_{tw} = 0$ implies $MPt_m = MPt_w$ so that

$$z_m \gamma L = t_m z_m \theta z_w \rightarrow t_m = \frac{\gamma L}{\theta z_w}. \tag{B.1}$$

Combining the FOC for MPK and MPn_d, gives

$$(z_m t_m)^{\gamma} \left(\frac{w \, \nu \, L}{(r + \delta) \, \theta} \right)^{\nu} \theta L^{\theta-1} = w. \tag{B.2}$$

Combining (B.1)-(B.2) gives

$$L^{\gamma+\theta+\nu-1} \left(\frac{z_m \gamma}{\theta z_m} \right)^{\gamma} \left(\frac{w\nu}{\theta(r + \delta)} \right)^{\nu} \theta = w, \tag{B.3}$$

which is false in general given that $\gamma + \theta + \nu - 1 = 0$. We conclude that if the borrowing constraint does not bind, then an employer optimally choose to devote all his time to managerial tasks.

Step 2: Assume that the borrowing constraint binds ($k = \overline{k}$). We now show that there exists a threshold level of assets $a^*(z_m, z_w)$ such that the optimal production plan features $t_w > 0$ if $a < a^*(z_m, z_w)$ and $t_w = 0$ if $a > a^*(z_m, z_w)$. Thus, if the borrowing constraint is not too tight, employers allocate all their time to managerial activities. We now find conditions for which $t_m < 1 (or, equivalently, t_w > 0)$. Note that $t_m < 1$ only if $\mu_{tw} = 0$. In this case, the marginal product of entrepreneurial time is equated across the two uses of time. From the FOC it can be obtained that

$$MPtw = MPtm \Rightarrow L = \frac{\theta z_w t_m}{\gamma}.$$

Plugging L into the FOC with respect to labor demand and solving for t_m gives an expression for the optimal fraction of time dedicated to managerial tasks:

$$t_m = \left[\frac{\theta z_m^{\gamma} \overline{k}^{\nu}}{w} \left(\frac{\gamma}{z_w \theta} \right)^{1-\theta} \right]^{\frac{1}{1-\gamma-\theta}}.$$

Note that $t_m < 1$ iff

$$\overline{k}(a, z_m, z_w) < k^*(z_m, z_w) \equiv \left[\frac{w}{\theta z_m^{\gamma}} \left(\frac{z_w \theta}{\gamma} \right)^{1-\theta} \right]^{\frac{1}{\nu}}.$$

Since $\overline{k}(a, z_m, z_w)$ is increasing in a, the inverse of this function can be used to define a threshold level of assets $a^*(z_m, z_w)$ such that $t_m < 1$ if and only if assets are below this threshold. Otherwise, $t_m = 1$.

Step 3: Compute the marginal product of employers time. From Step 1 and 2, when assets are below $a^*(z_m, z_w)$ we have $MPtm = MPtw = wz_w$. On the other hand, when assets are above $a^*(z_m, z_w)$, $t_m = 1$ and $MPtm > MPtw$. To obtain an expression for MPtm note that the FOC with respect to capital and outside labor imply:

$$\begin{aligned} k &= \frac{w\nu}{(r + \delta + \mu_k)\theta} n_d \\ n_d &= \left(\frac{\theta z_m^{\gamma}}{w} \left[\frac{w\nu}{(r + \delta + \mu_k)\theta} \right]^{\nu} \right)^{\frac{1}{1-(\nu+\theta)}} \end{aligned}$$

Plugging k and n_d into $MPtm = \gamma z_m^{\gamma} k^{\nu} n_d^{\theta}$ gives

$$MPt_m = z_m \gamma \left[\left(\frac{\nu}{(r + \delta + \mu)} \right)^{\nu} \left(\frac{\theta}{w} \right)^{\theta} \right]^{\frac{1}{1-(\nu+\theta)}}$$

Proof of Proposition 5. An individual with ability (z_m, z_w) and assets a prefers being an employer rather than self-employment if and only if

$$\left(z_m^{\gamma} z_w^{\theta} \right)^{\frac{1}{\theta+\gamma}} r_{mw} + \mu_{se} k_{se} + ra < z_m r_m + \mu_e k_e + ra,$$

where μ_e and μ_{se} are the Lagrange multipliers associated to the borrowing constraints when the individual is an employer or is self-employed, respectively, and k_e and k_{se} are the capital used in production at these occupations. This inequality holds when the ability ratio is such that

$$\left[\frac{r_m}{r_{mw}} + \frac{(\mu_e k_e - c_f)}{z_m r_{mw}} - \frac{(\mu_{se} k_{se})}{z_m r_{mw}} \right]^{\frac{\theta+\gamma}{\theta}} \equiv \overline{R_2},$$

If capital markets are perfect ($\phi = 1$), the Lagrange multiplier on the borrowing constraint is equal to zero ($\mu = 0$) and the individual prefers to be an employer rather than be self-employed if and only if

$$\frac{z_w}{z_m} < R_2 \left(1 - \frac{c_f}{z_m r_{mw}}\right)^{\frac{\theta+\gamma}{\theta}}, \text{ where } R_2 \equiv \left(\frac{r_m}{r_{mw}}\right)^{\frac{\theta+\gamma}{\theta}}.$$

Proof of Proposition 6. When $\phi = 1$ the Lagrange multiplier on the borrowing constraint is equal to zero ($\mu = 0$) and occupational choice decisions are independent of asset holdings and maximize the marginal product of time. Proposition 4 established that an individual with ability (z_m, z_w) prefers to be self-employed rather than work for a wage if and only if the skill ratio satisfies

$$\frac{z_w}{z_m} < R_1 \equiv \left(\frac{r_{mw}}{w}\right)^{\frac{\theta+\gamma}{\gamma}}.$$

Assuming that the fixed cost of operation $c_f = 0$, then Proposition 5 implies that an individual with ability (z_m, z_w) prefers to be entrepreneur instead of self-employed if and only if the skill ratio satisfies

$$\frac{z_w}{z_m} < R_2 \equiv \left(\frac{r_m}{r_{mw}}\right)^{\frac{\theta+\gamma}{\theta}}.$$

If parameters are such that $R_2 < R_1$, then self-employment is dominated by either being an employer or a worker. The optimal occupational choice is to be an employer if and only if $\frac{z_m}{z_w} > \frac{w}{r_m}$. Otherwise, the optimal occupational choice is to work for a wage. When employers incur a fixed cost of operation, being an employer is preferred to being self-employed if and only if

$$\left(z_m^{\gamma} z_w^{\theta}\right)^{\frac{1}{1-\nu}} r_{mw} < (z_m r_m - c_f),$$

which holds when $\frac{z_w}{z_m} < \left[\frac{r_m}{r_{mw}} - \frac{c_f}{r_{mw} z_m}\right]^{\frac{\theta+\gamma}{\theta}}$.

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